Time-varying networks

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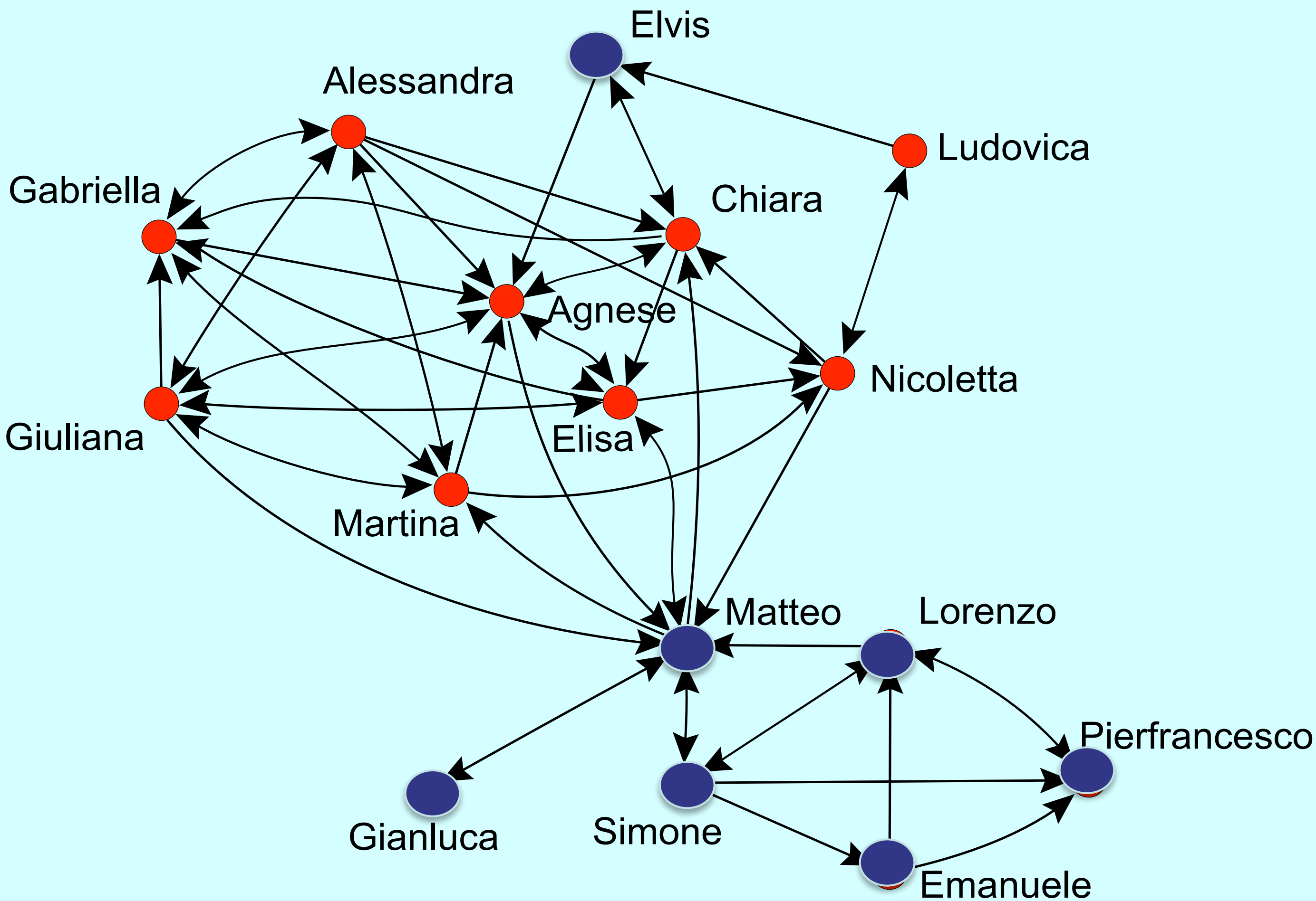
Complex networks



Friendships at the kindergarten of my daughter Elisa (2006)

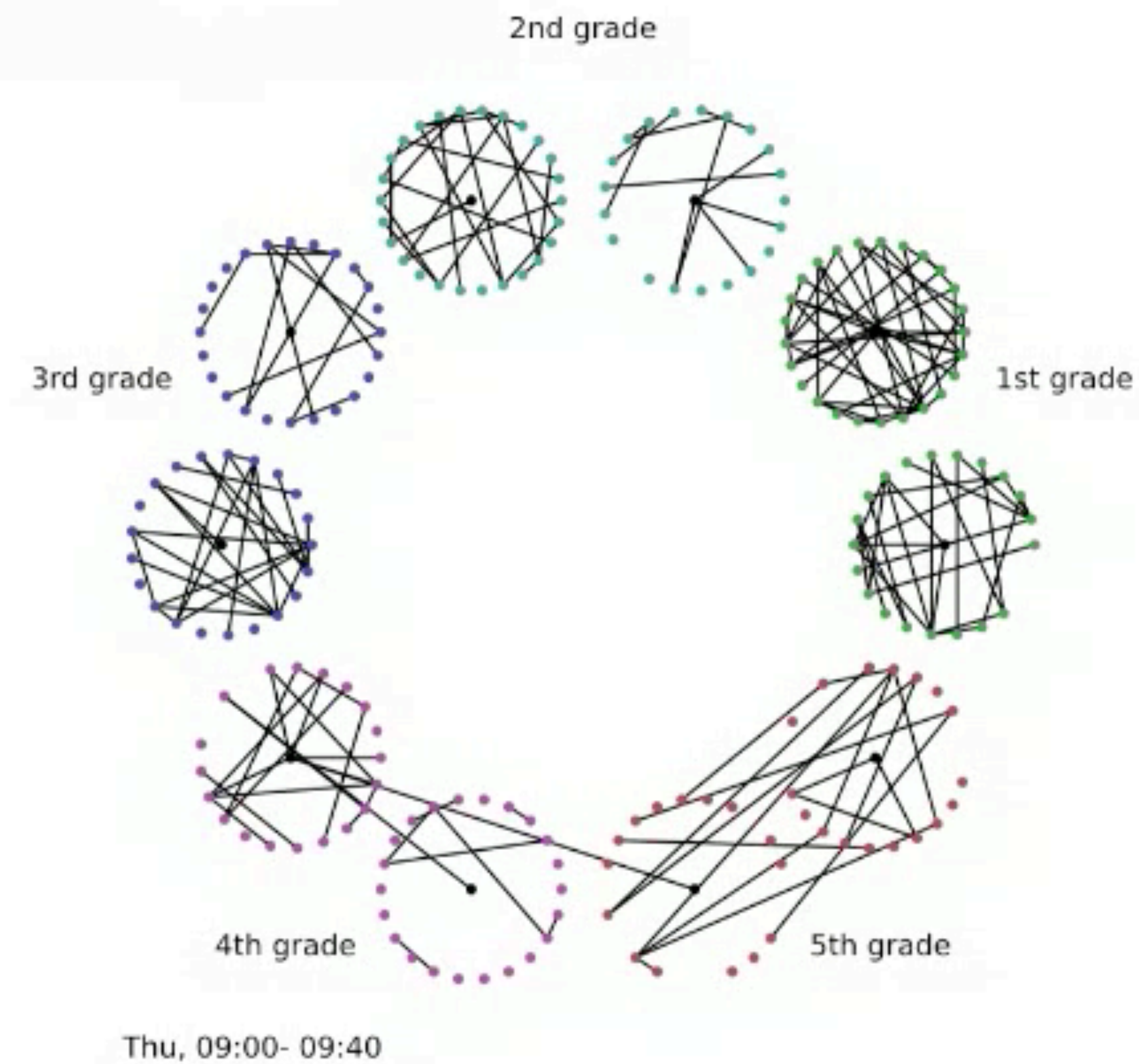
One school day: contacts over time in a French primary school

Stehle et al. PLoS ONE 6, e23176 (2011)

-- 10 classes

-- 232 children,
10 teachers

-- face-to-face contacts.
Radio frequency proximity-sensing (<1.5 m every 20 sec)



MIT Reality Mining

Eagle, Pentland
Pers Ubiquit Comput (2006)

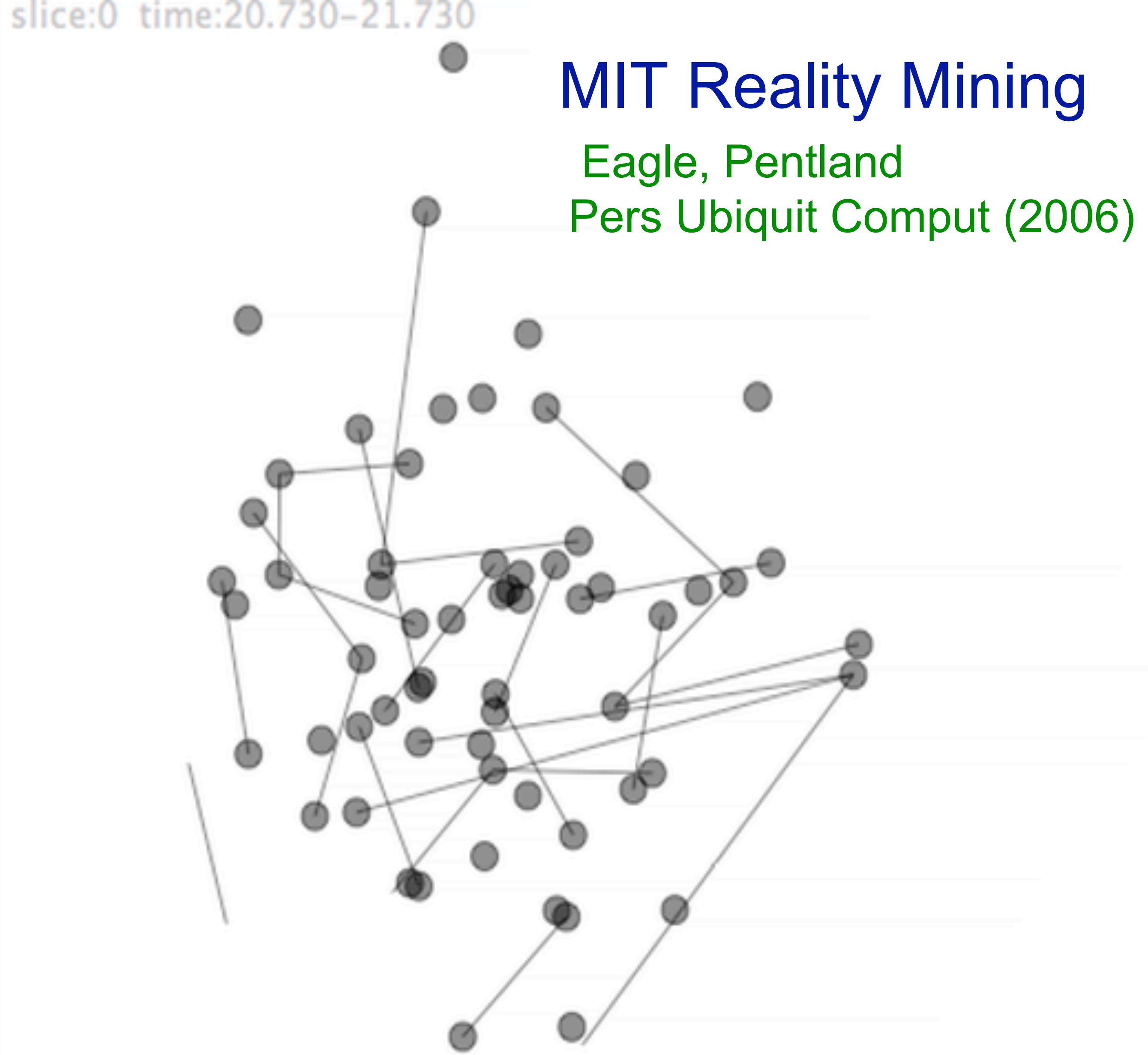
-- 9 months of interactions

-- 100 MIT students and faculties

-- bluetooth devices

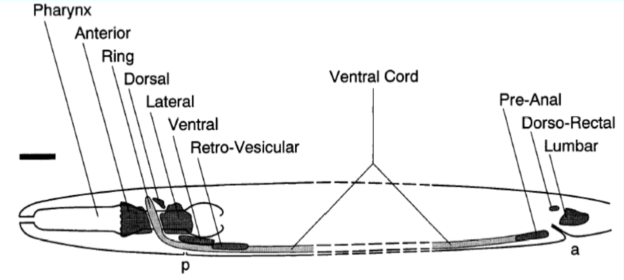
-- every 5 minutes

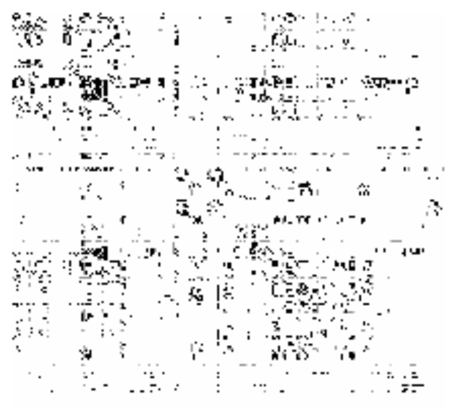
INFOCOM 2006
CAMBRIDGE 2010



Brain Networks

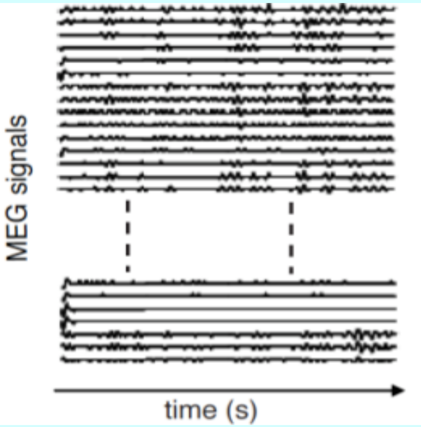
C. elegans: layout of ganglia

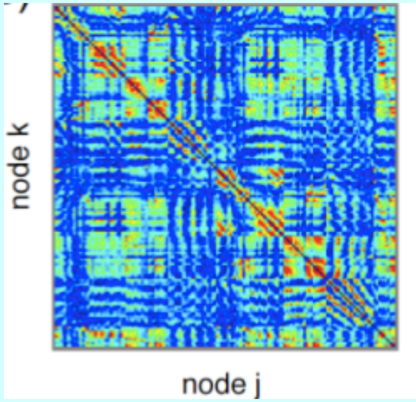




Brenner et al, 1975 mapped every single nervous cell.

Anatomical connectivity





EEG, MEG, fMRI signals

Coherence, synchronization, causality

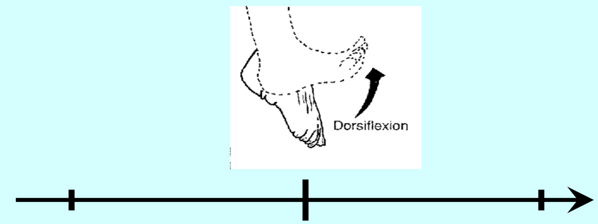
Functional connectivity

Cortical networks over time during a task

Physical Tasks:

Foot movement

61 EEG channels cap



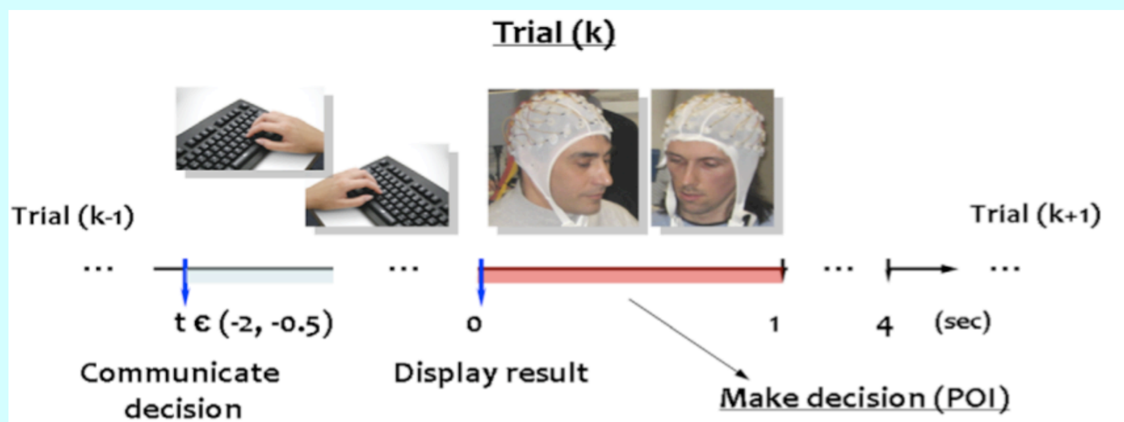
- 1 s **onset** **+ 1 s**

Preparation → Execution

De Vico Fallani, Latora, et al.,
J. Phys. A 41, 224014 (2008)

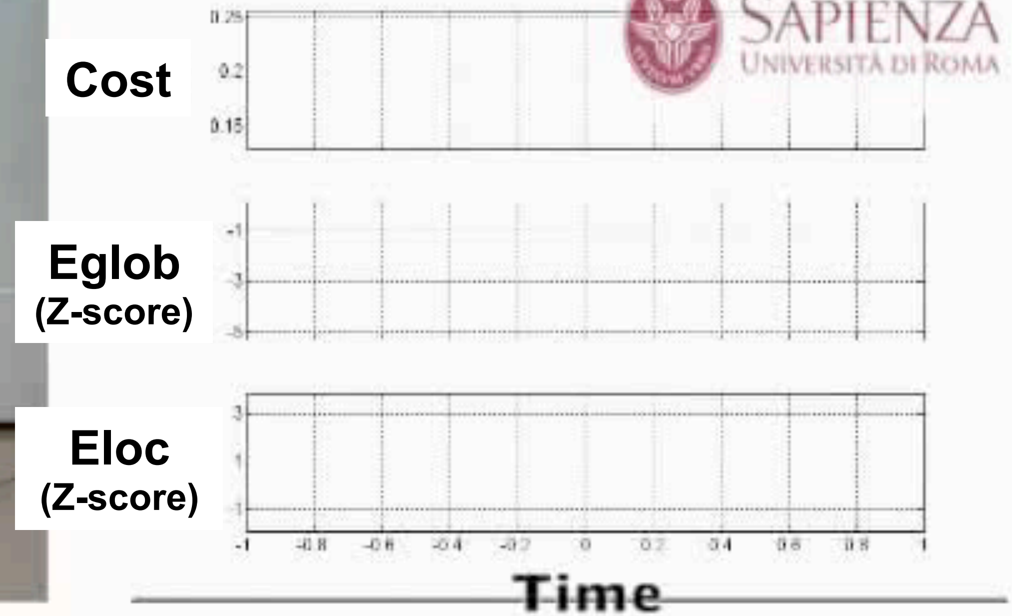
Social Tasks:

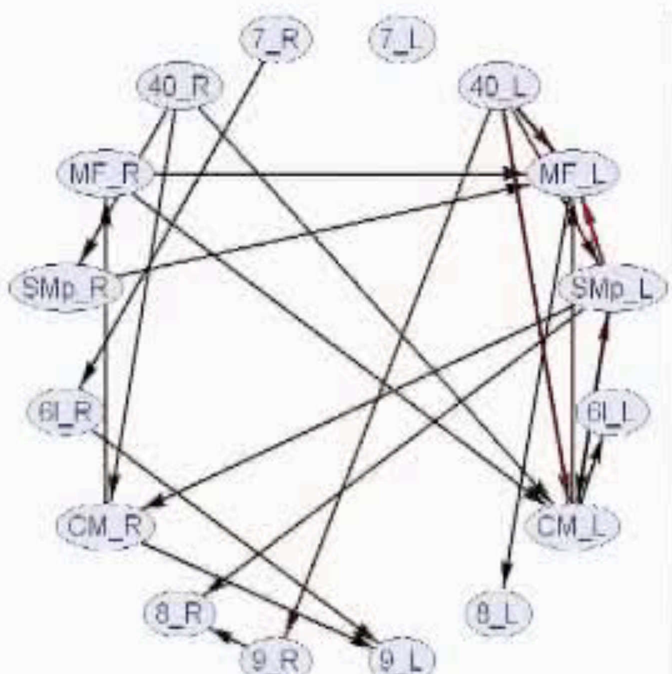
Two individuals playing the iterated prisoner's dilemma

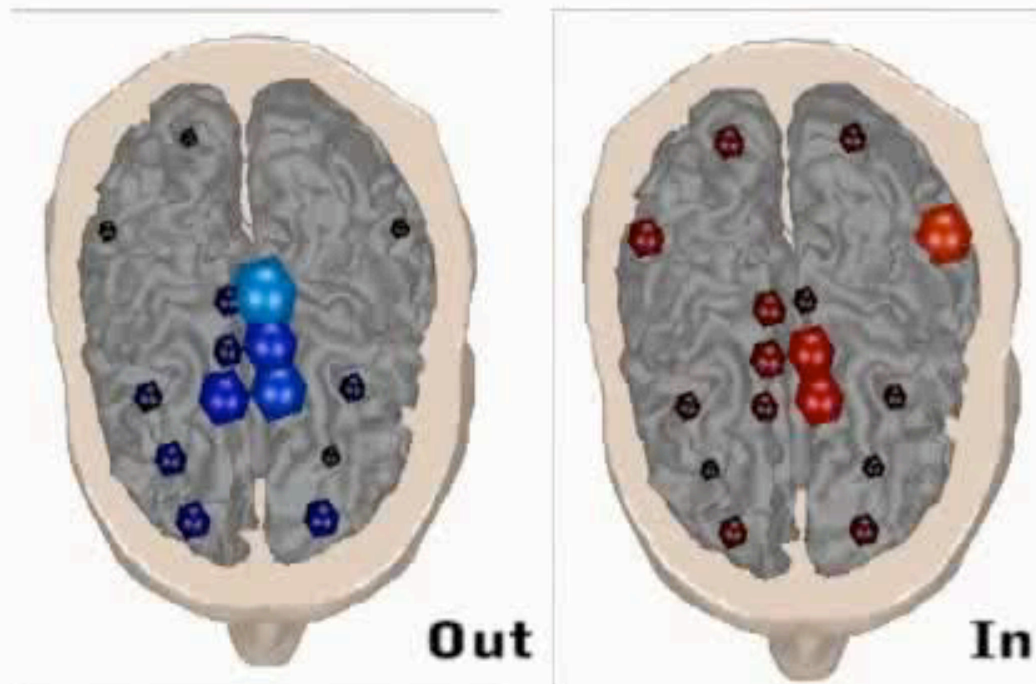


De Vico Fallani et al. PLoS ONE 2011

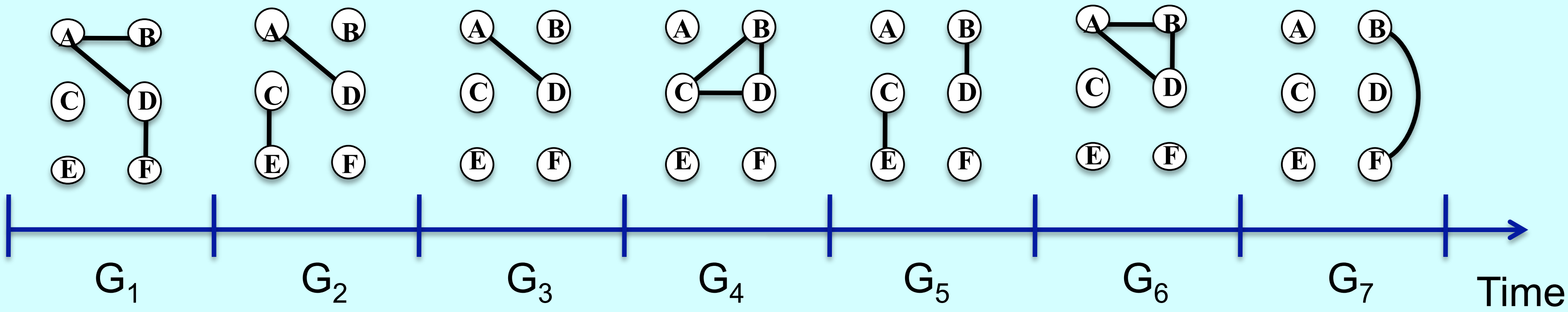
SAPIENZA
Università di Roma

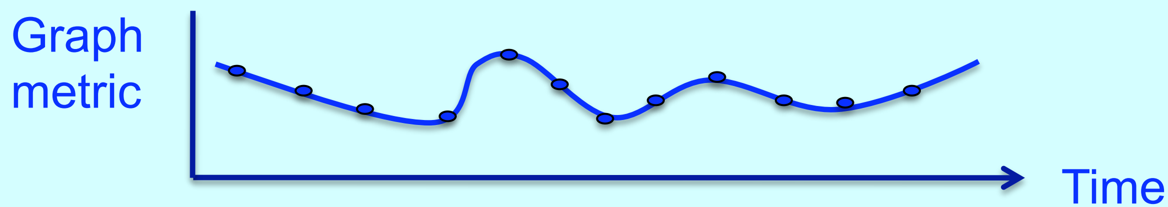




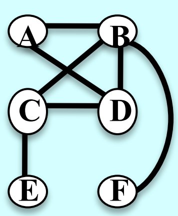


Time ordered sequence of graphs

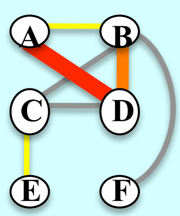




How to characterize the entire sequence as a whole?



Aggregated

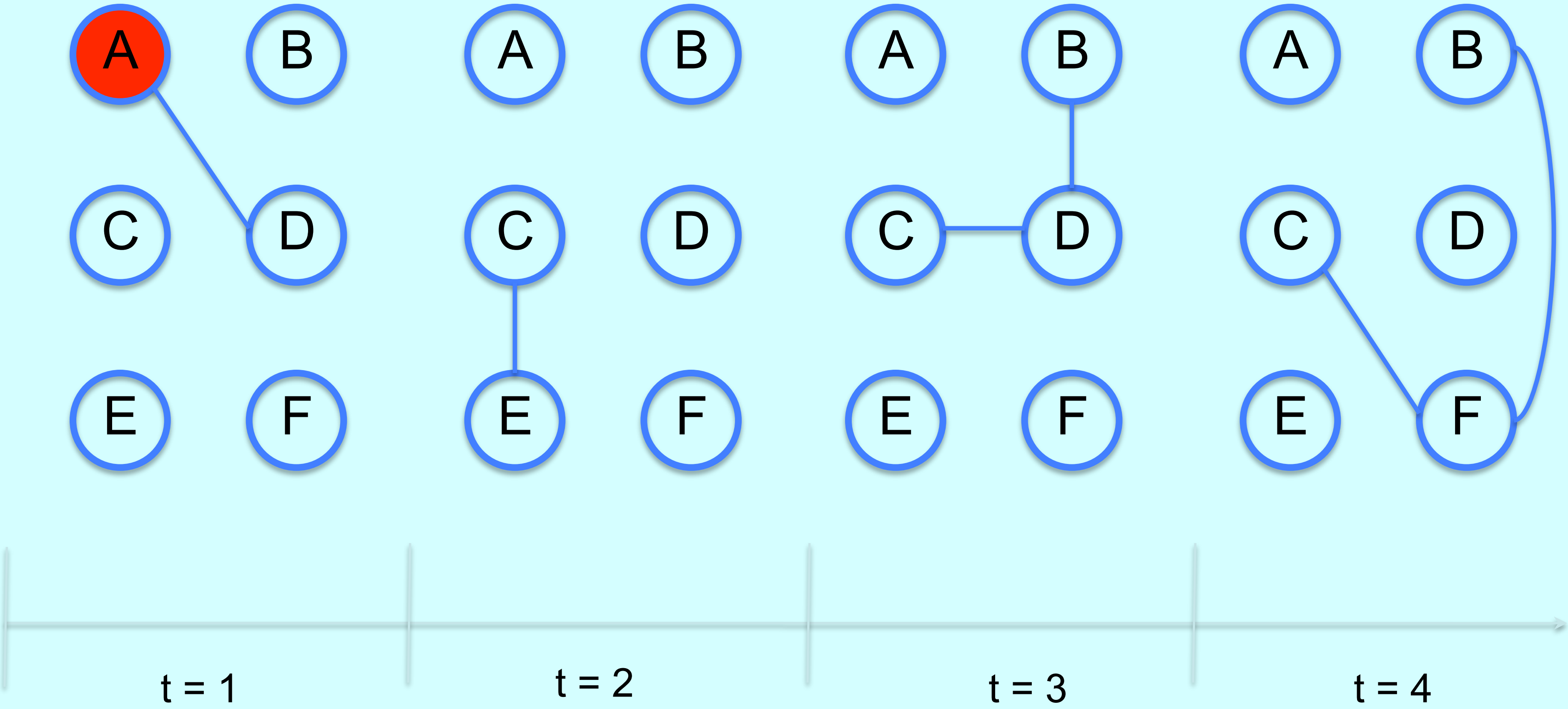


Weighted aggregated

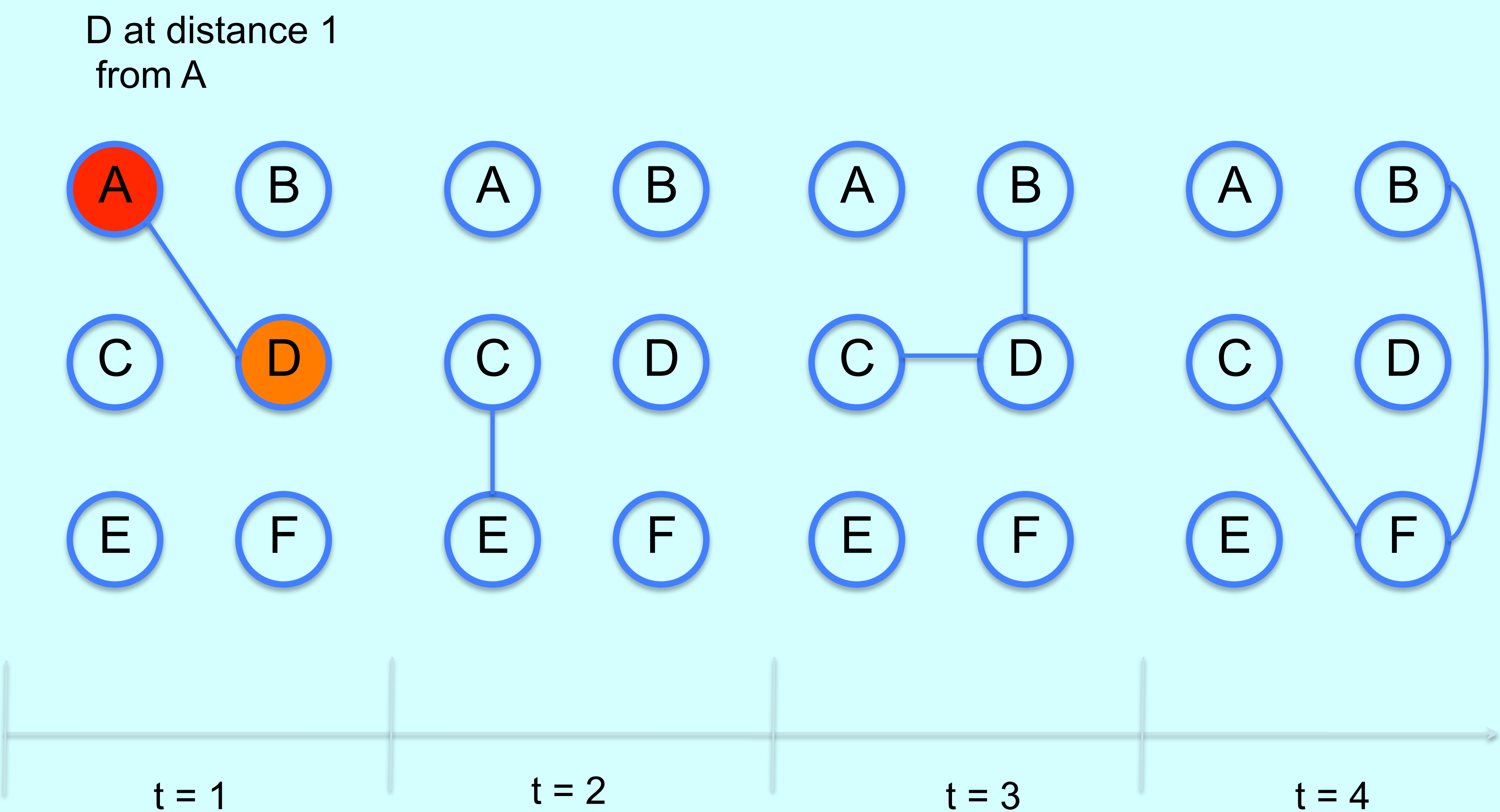
We miss time order and correlations between links

New metrics !!

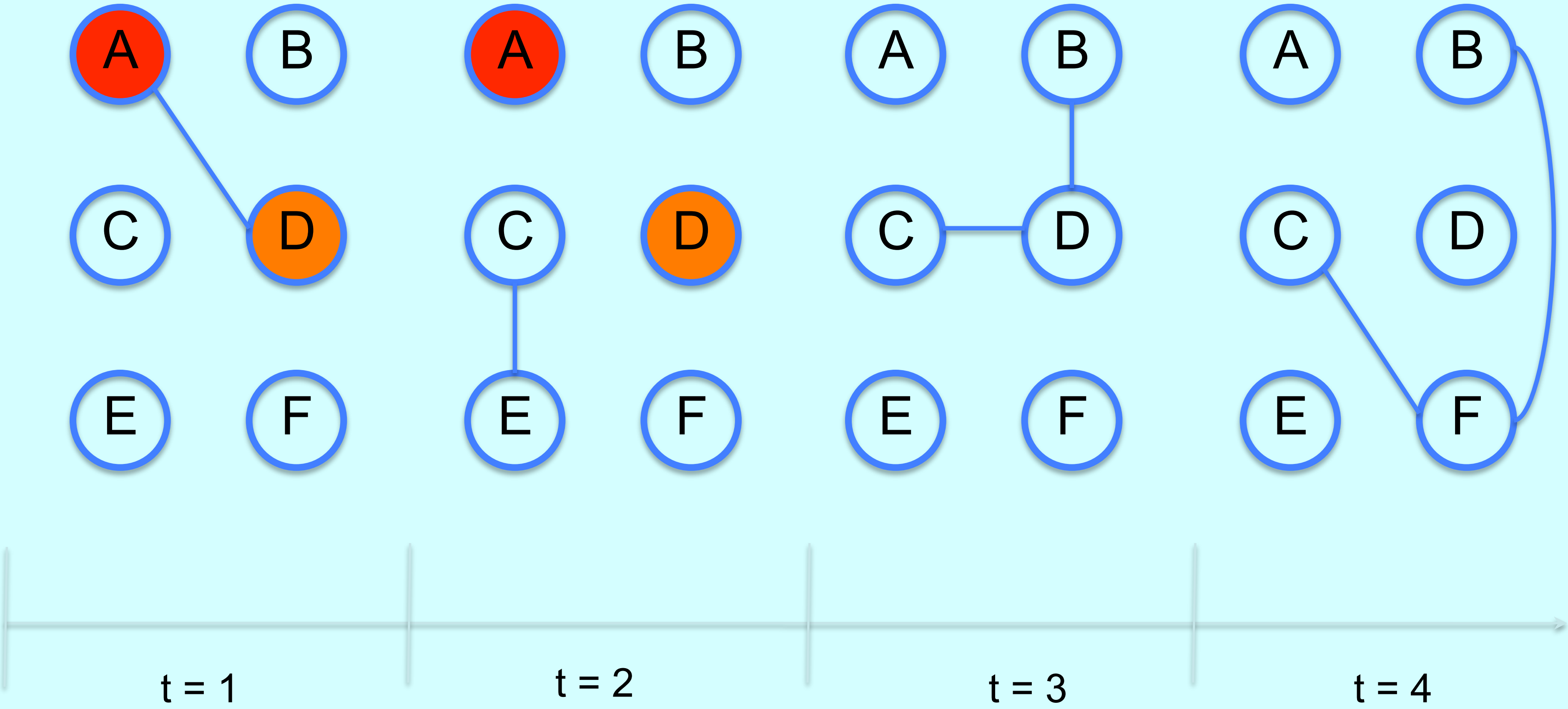
Temporal paths and distances in TVN



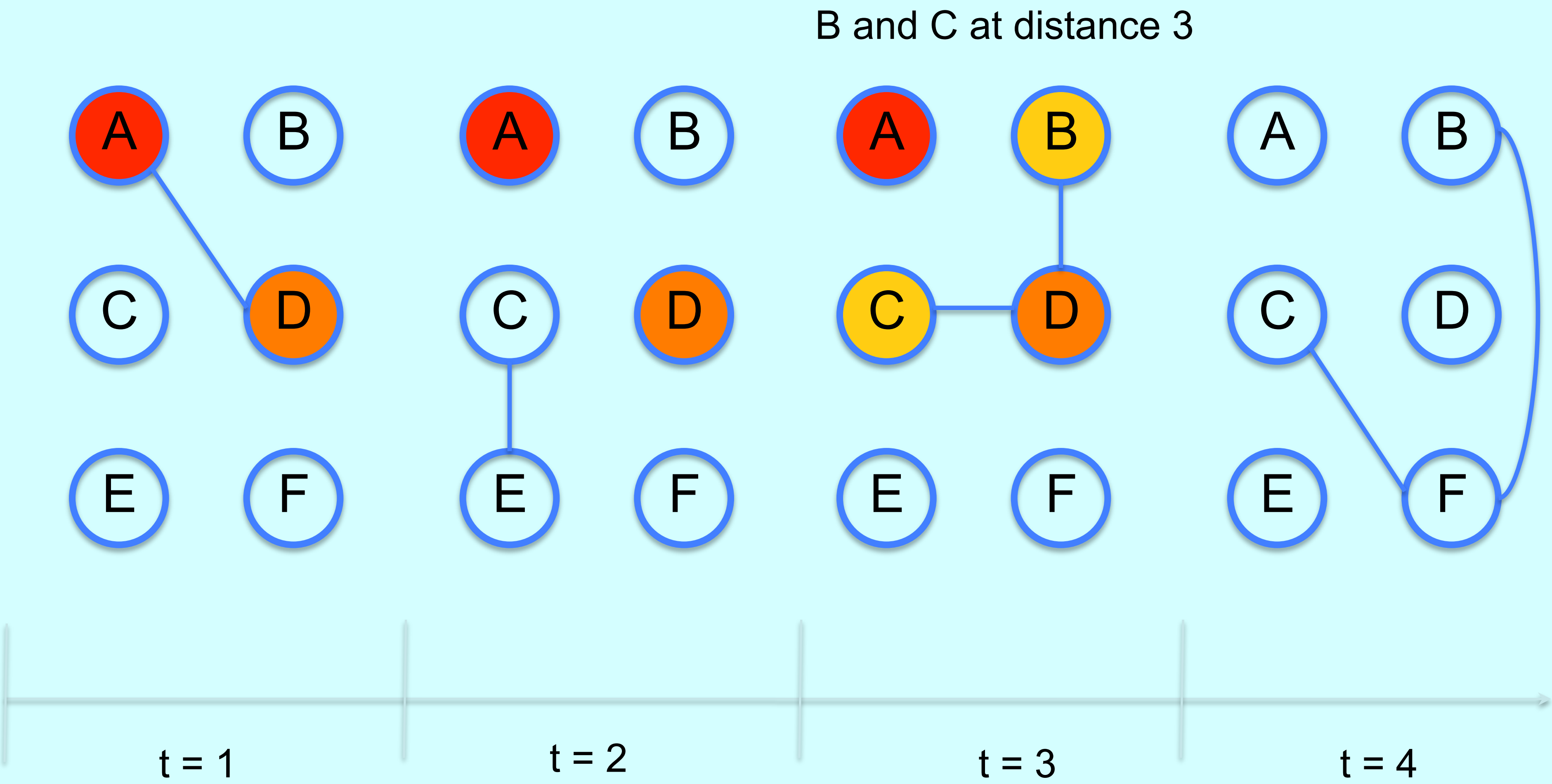
Temporal paths and distances in TVN



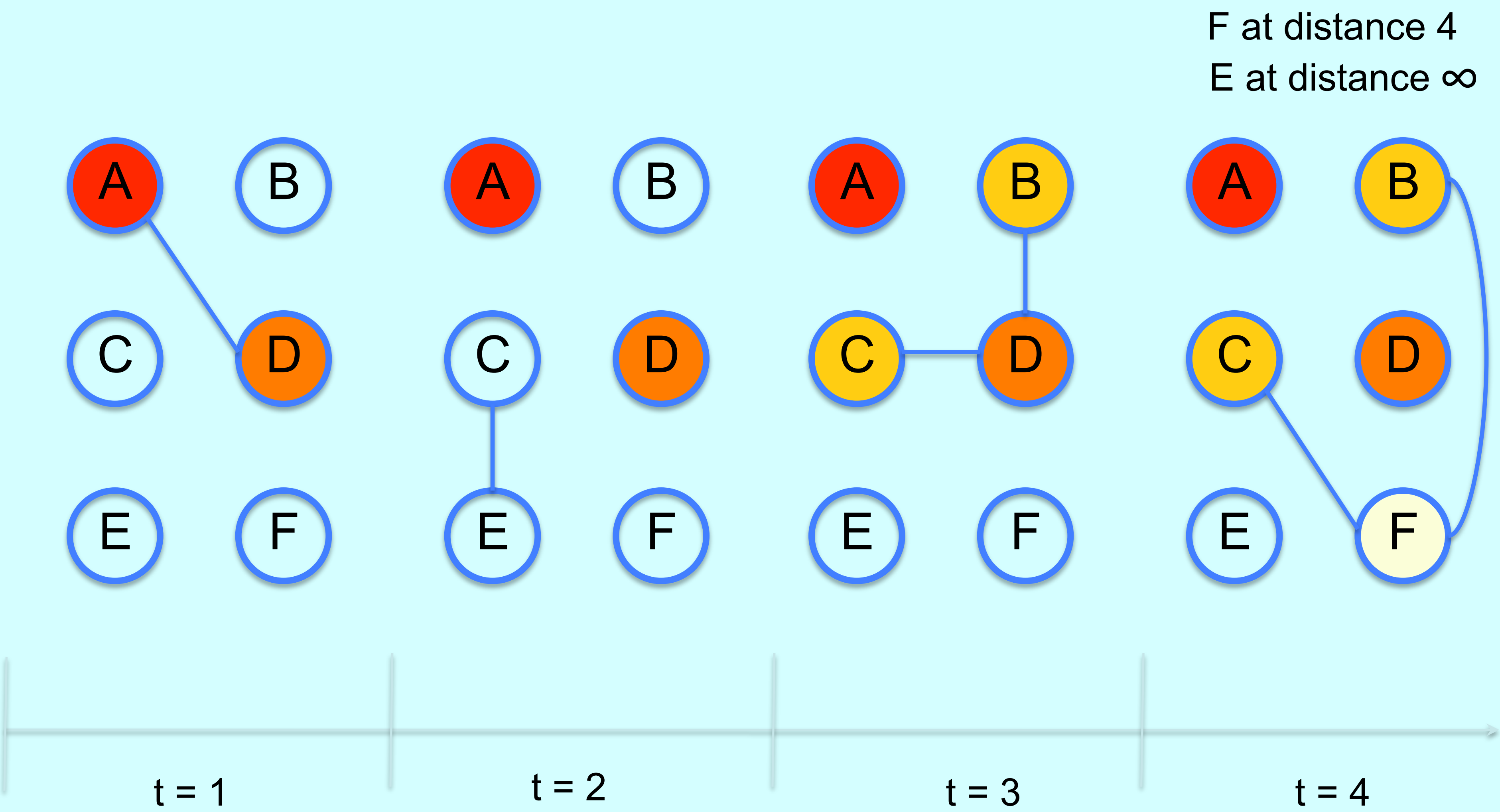
Temporal paths and distances in TVN



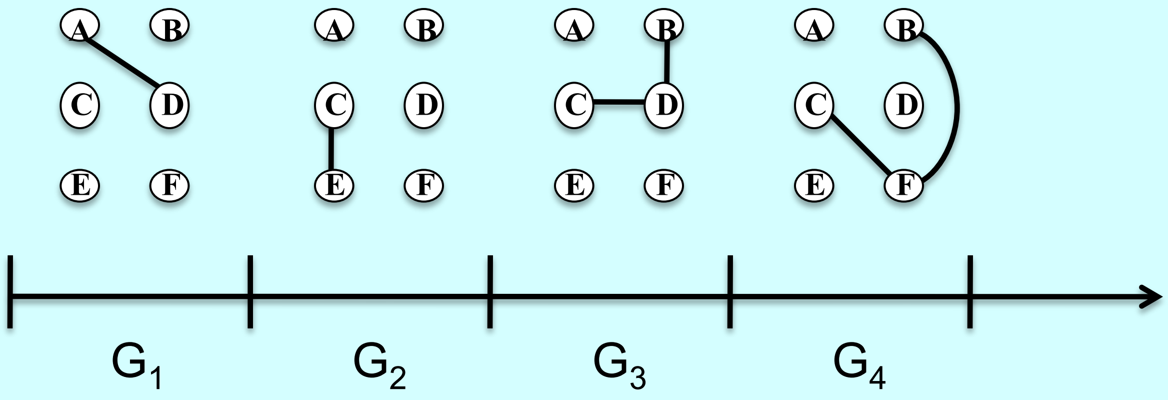
Temporal paths and distances in TVN



Temporal paths and distances in TVN



Directionality introduced by time



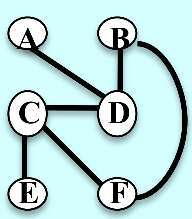
Path A → F

No path F → A

Path E → D + path D → A

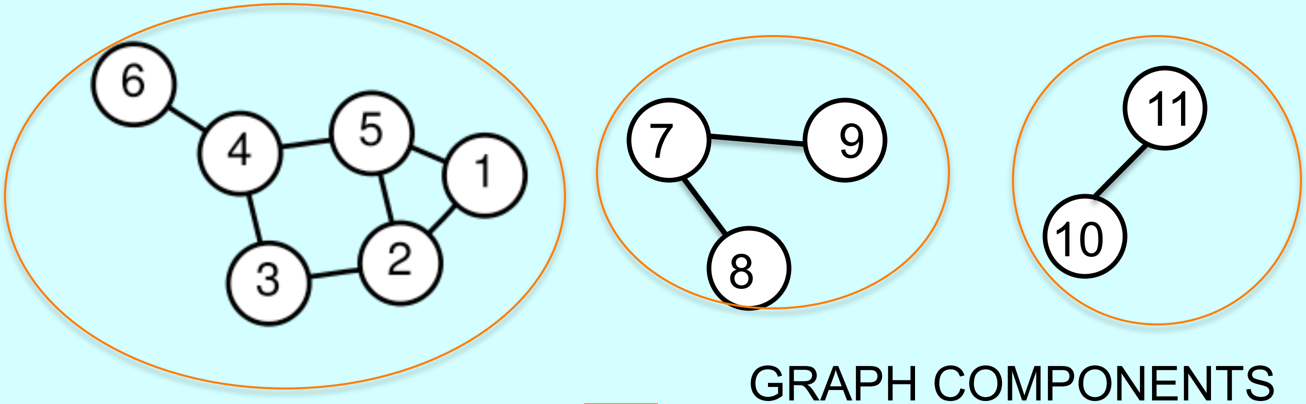
No path E → A

AGGREGATE
STATIC NETWORK



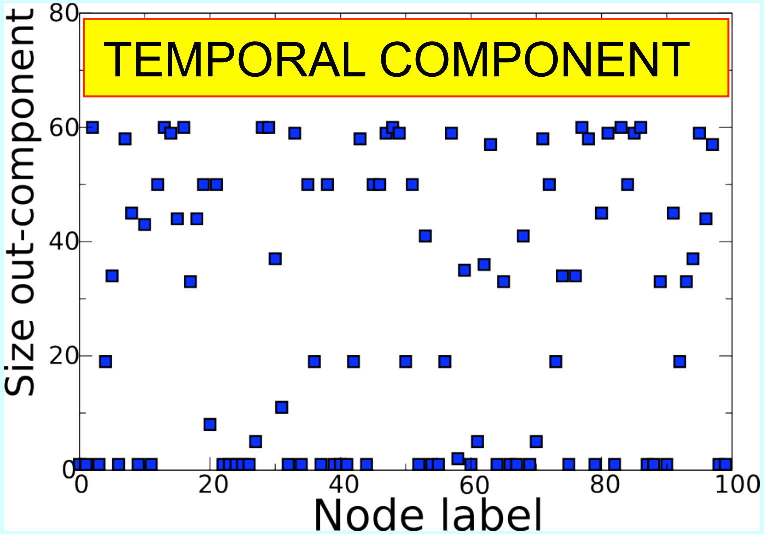
Paths are SYMMETRIC and TRANSITIVE

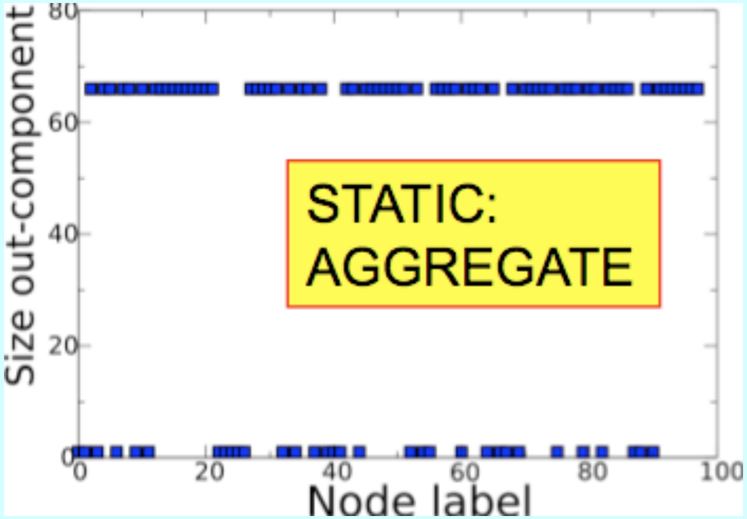
Extension of basic concepts



GRAPH COMPONENTS
IN A STATIC GRAPH

One week of MIT Reality Mining





Nicosia, Tang, Musolesi, Russo, Mascolo, Latora, CHAOS 22, 023101 (2012)

Exploit time in def of centrality measures

Standard centrality measures (closeness, betweenness…) can be generalized to take into account of allowed temporal paths


The Physics arXiv Blog

technology review

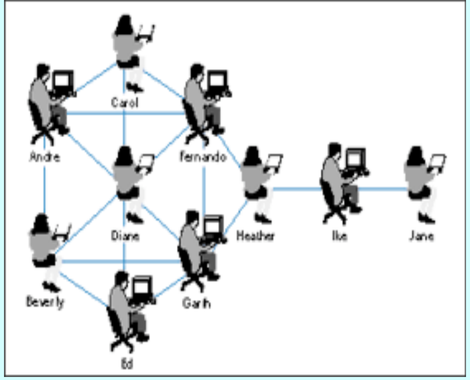
Published by MIT

White Worm Could Stop Bluetooth Viruses

Viruses that spread by Bluetooth or WiFi could be completely contained by a new type of white worm, say computer security researchers

KFC 12/08/2010

Tang, Mascolo, Musolesi, Latora,
WoWMoM11 and arXiv: 1012.0726 (2010)



Centrality measures

Closeness

Sabidussi 1966

$$C_i = \frac{N-1}{\sum_j d_{ij}}$$

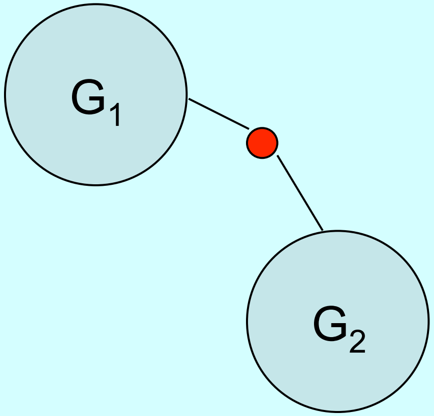
Central nodes are close to other graph nodes

Betweenness

Freeman 1977

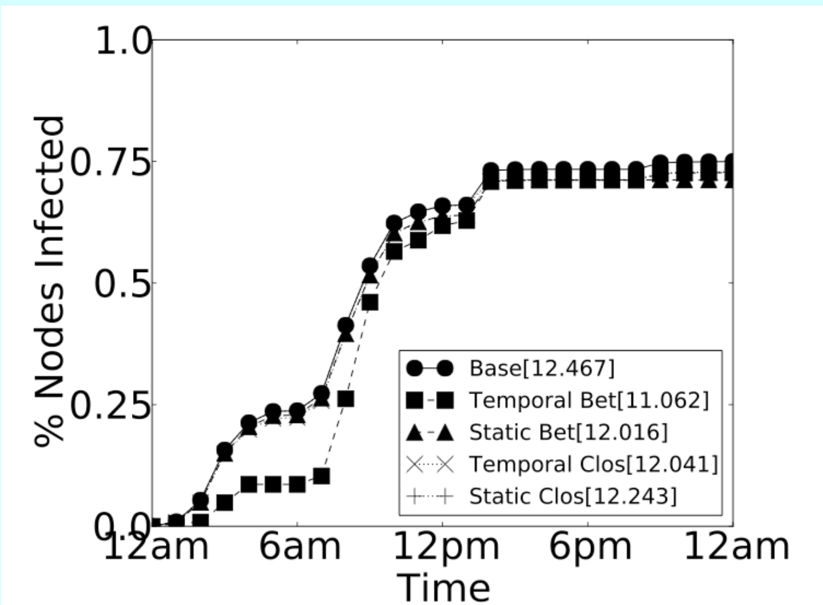
$$B_i = \frac{1}{(N-1)(N-2)} \sum_j \sum_k \frac{n_{jk}(i)}{n_{jk}}$$

Central nodes are in between other graph nodes

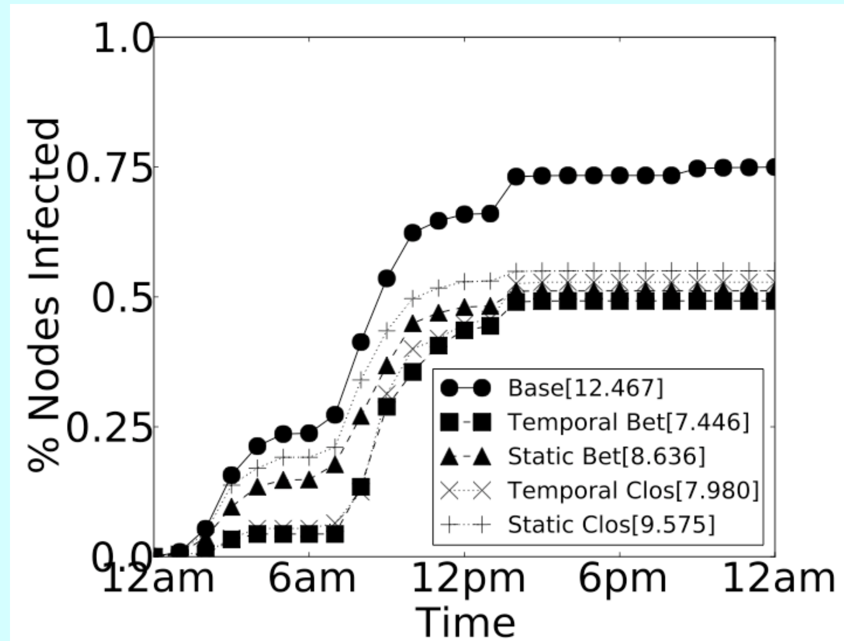


Exploit time in def of centrality measures

A person receives on his/her devices a malicious message (virus) in the morning, which replicates to any device met during the day



Immunization of the node with the highest centrality

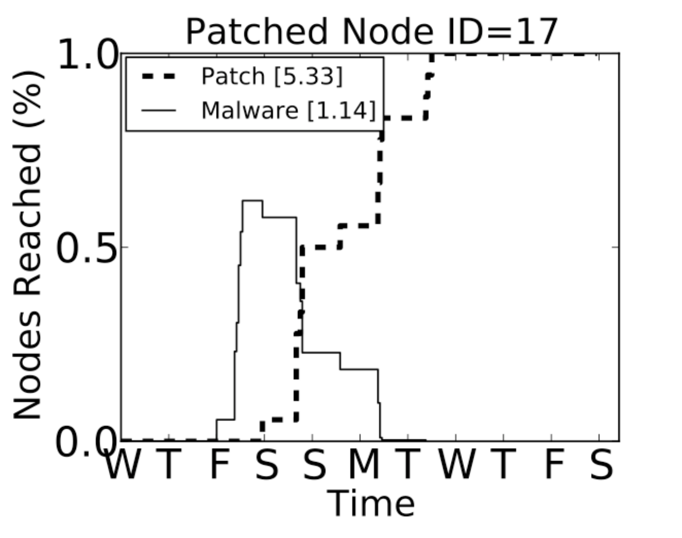


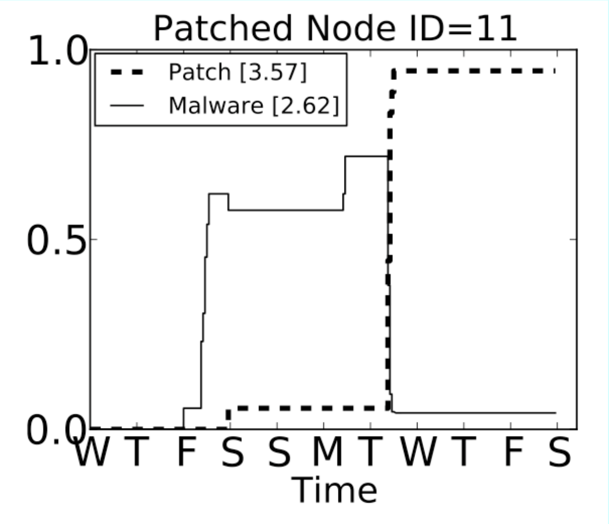
Immunization of the 10 nodes with the highest centrality

Tang et al, arXiv: 1012.0726

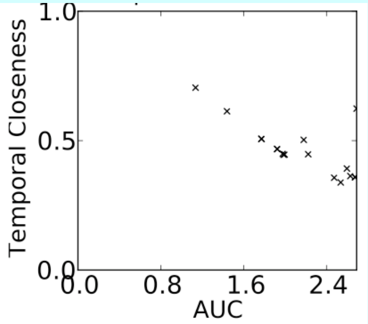
Spreading of a patch message

Worst case scenario: $[t_m\text{=Fri 12am}, \; t_p\text{=Sat 12am}]$





Patch is started at the node
with highest temporal closeness



Tang et al, arXiv: 1012.0726

Small-world behavior in TVG ?

1) How good is the communication between nodes in a TVG?

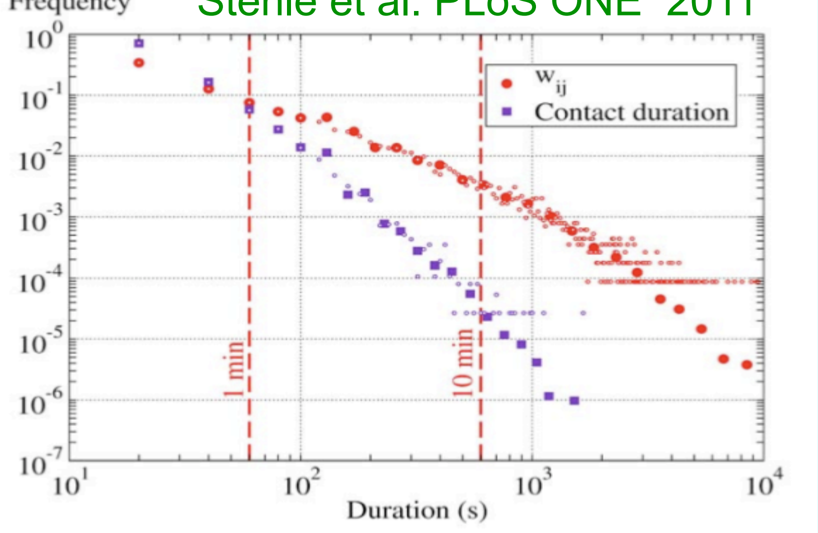
Average temporal distance

$$L = \frac{1}{N(N-1)} \sum_{ij} d_{ij}$$

Tang, Musolesi, Mascolo, Latora, ACM SIGCOMM (2009)

2) Time persistence of links

Stehle et al. PLoS ONE 2011



Coefficient of temporal clustering

$$C = \frac{\sum_i C_i}{N}$$

$$C_i = \frac{1}{T-1} \sum_{t=1}^{T-1} \frac{\sum_j a_{ij}(t) a_{ij}(t+1)}{\sqrt{\left[\sum_j a_{ij}(t)\right]\left[\sum_j a_{ij}(t+1)\right]}}$$

Clauset, Eagle, DIMACS (2007)

Small-world behavior in TVG

L Typical temporal distance between 2 nodes

$0 \leq C \leq 1$ Temporal clustering: persistence of links

Null model: compare to temporally-shuffled sequences

Infocom 06

London net

	C	C^{rand}	L	L^{rand}	E	E^{rand}
α	0.44	0.18	3.9 (100%)	4.2 (98%)	0.50	0.48
β	0.40	0.17	6.0 (94%)	3.6 (92%)	0.41	0.45
γ	0.48	0.13	12.2 (86%)	8.7 (89%)	0.39	0.37
δ	0.44	0.17	2.2 (100%)	2.4 (92%)	0.57	0.56
$d1$	0.80	0.44	8.84 (61%)	6.00 (65%)	0.192	0.209
$d2$	0.78	0.35	5.04 (87%)	4.01 (88%)	0.293	0.298
$d3$	0.81	0.38	9.06 (57%)	6.76 (59%)	0.134	0.141
$d4$	0.83	0.39	21.42 (15%)	15.55(22%)	0.019	0.028
Mar	0.044	0.007	456	451	0.000183	0.000210
Jun	0.046	0.006	380	361	0.000047	0.000057
Sep	0.046	0.006	414	415	0.000058	0.000074
Dec	0.049	0.006	403	395	0.000047	0.000059

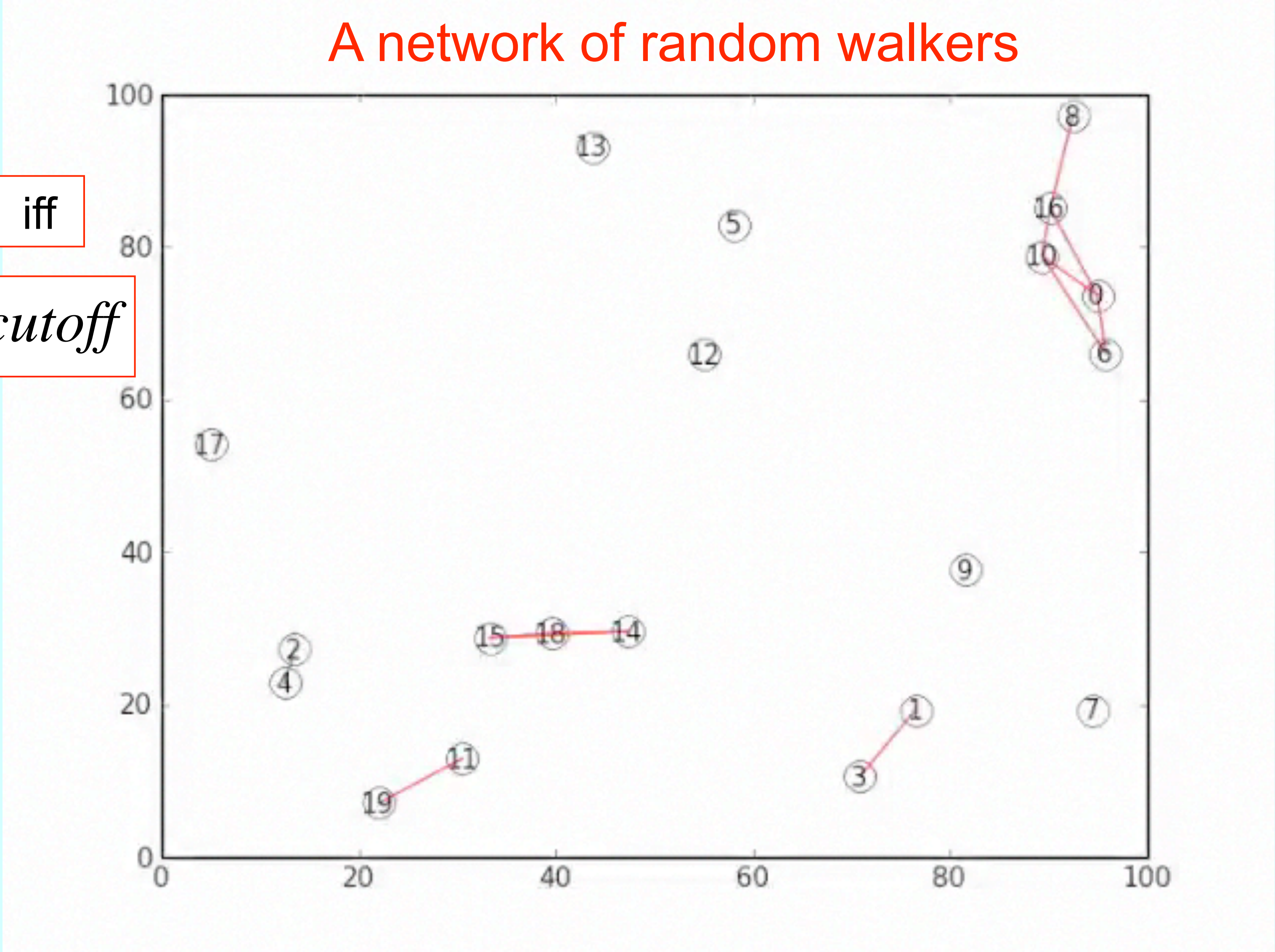
Real networks have L as small as in shuffled sequences, while C is much higher !!!

Tang, Scellato, Musolesi, Mascolo, Latora, Phys. Rev. E81, 055101R (2010)

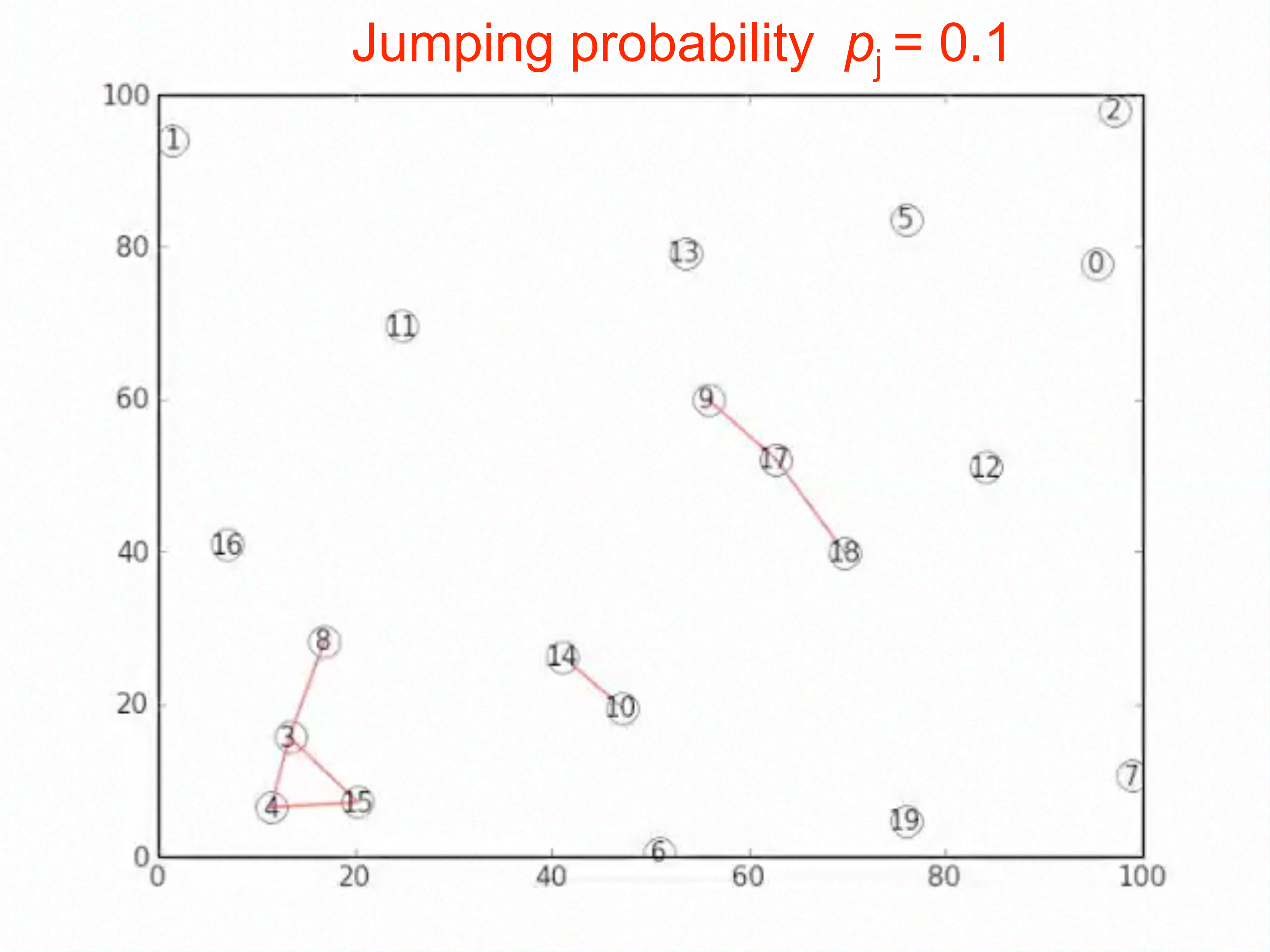
A simple model of temporal small worlds

Link (*i* , *j*) iff

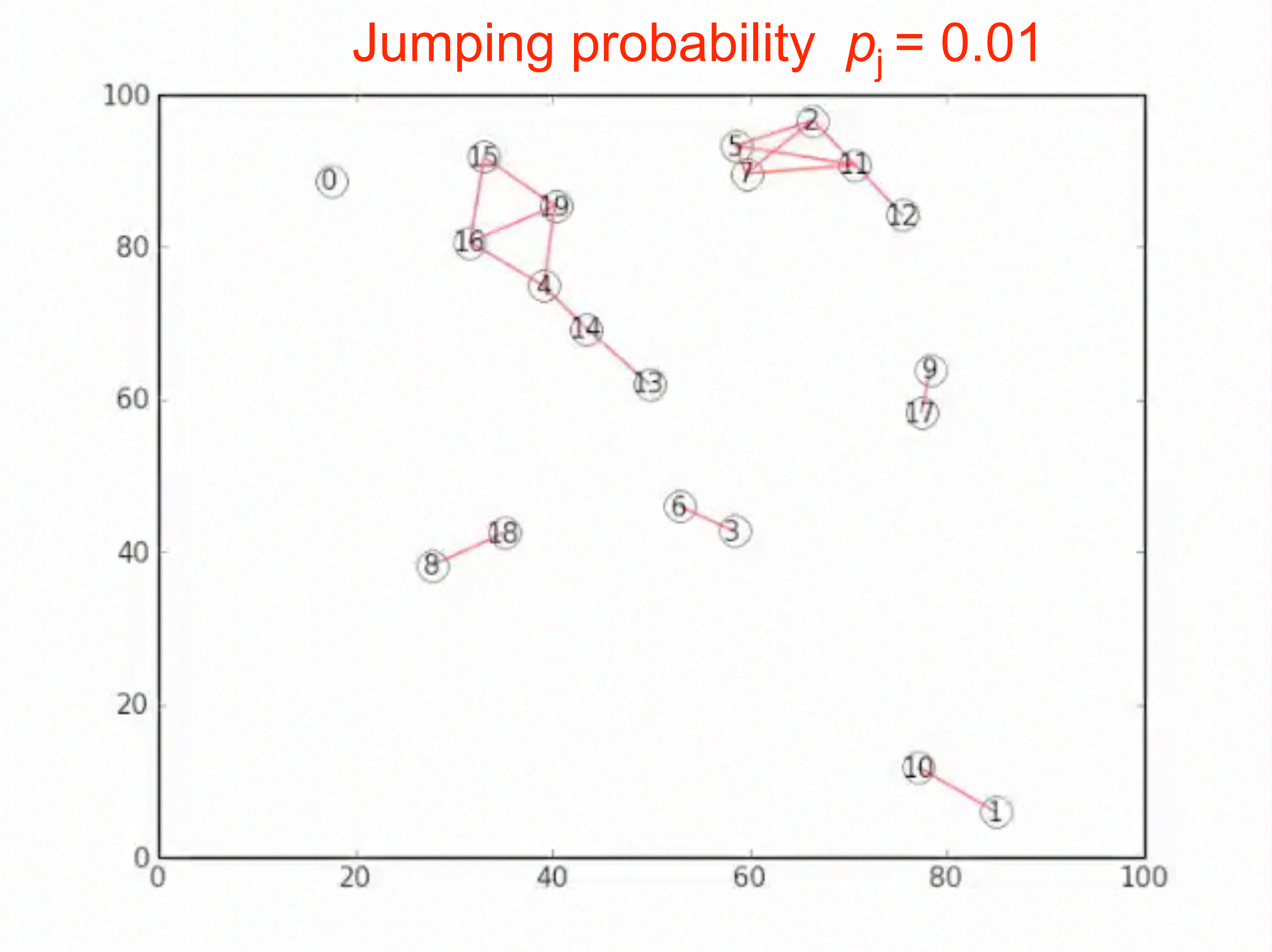
$$\left|\vec{r}_i - \vec{r}_j\right| < cutoff$$



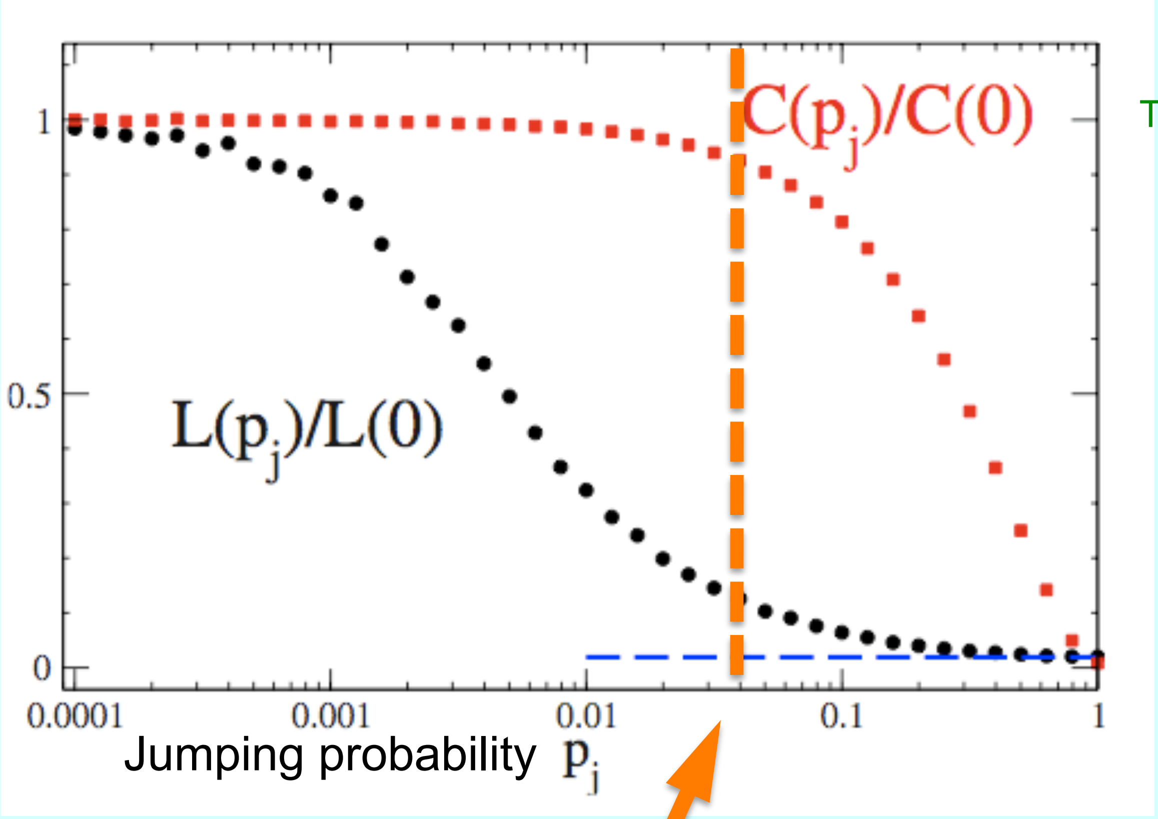
Random walkers with long-range jumps



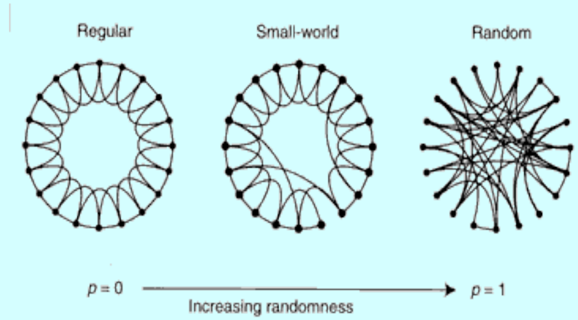
Random walkers with long-range jumps

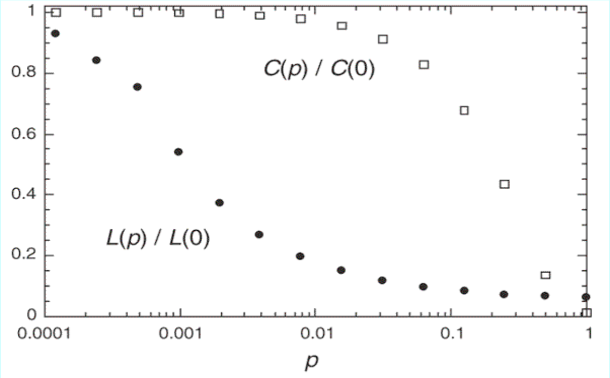


A simple model of temporal small worlds



Tang, Scellato, Musolesi, Mascolo, Latora, Phys. Rev. E81, 055101R (2010)





L as small as in shuffled sequences, while C is much higher !!!

Watts and Strogatz, Nature 339 (1998) 440

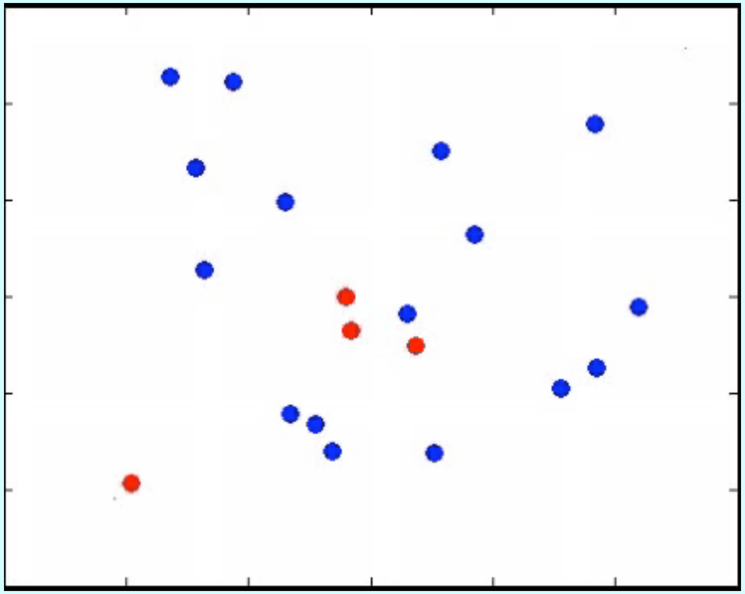
Epidemic spreading in temporal small worlds

Individuals can be in 3 states:

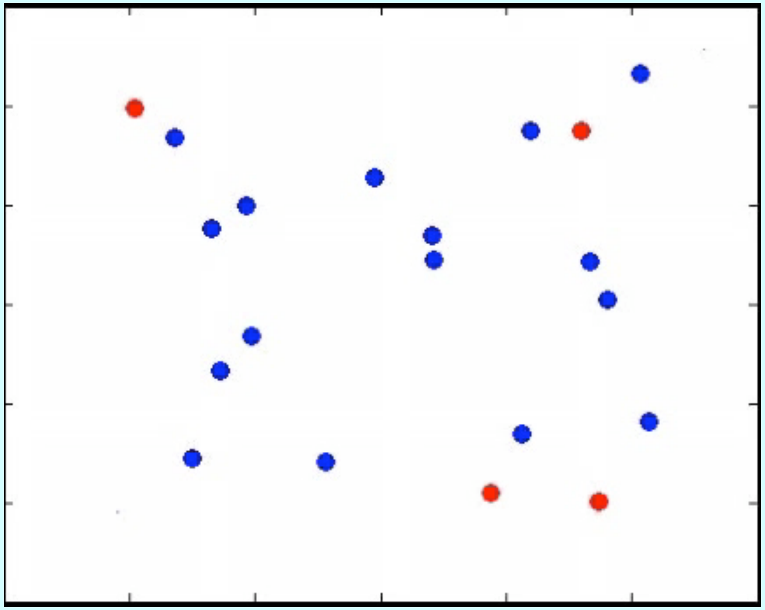
- -- Susceptible S
- -- Infective I
- -- Removed R

$$S + I \xrightarrow{\lambda} I + I$$

$$I \xrightarrow{\mu} R$$

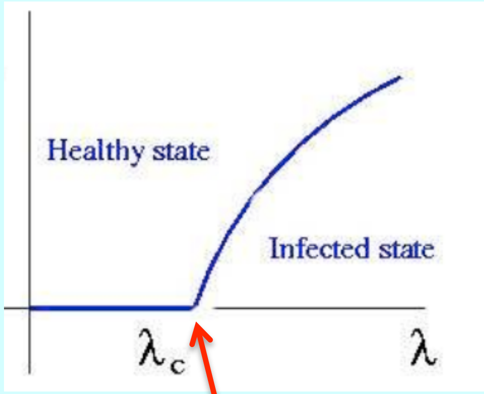


No jumps: $p_j = 0$

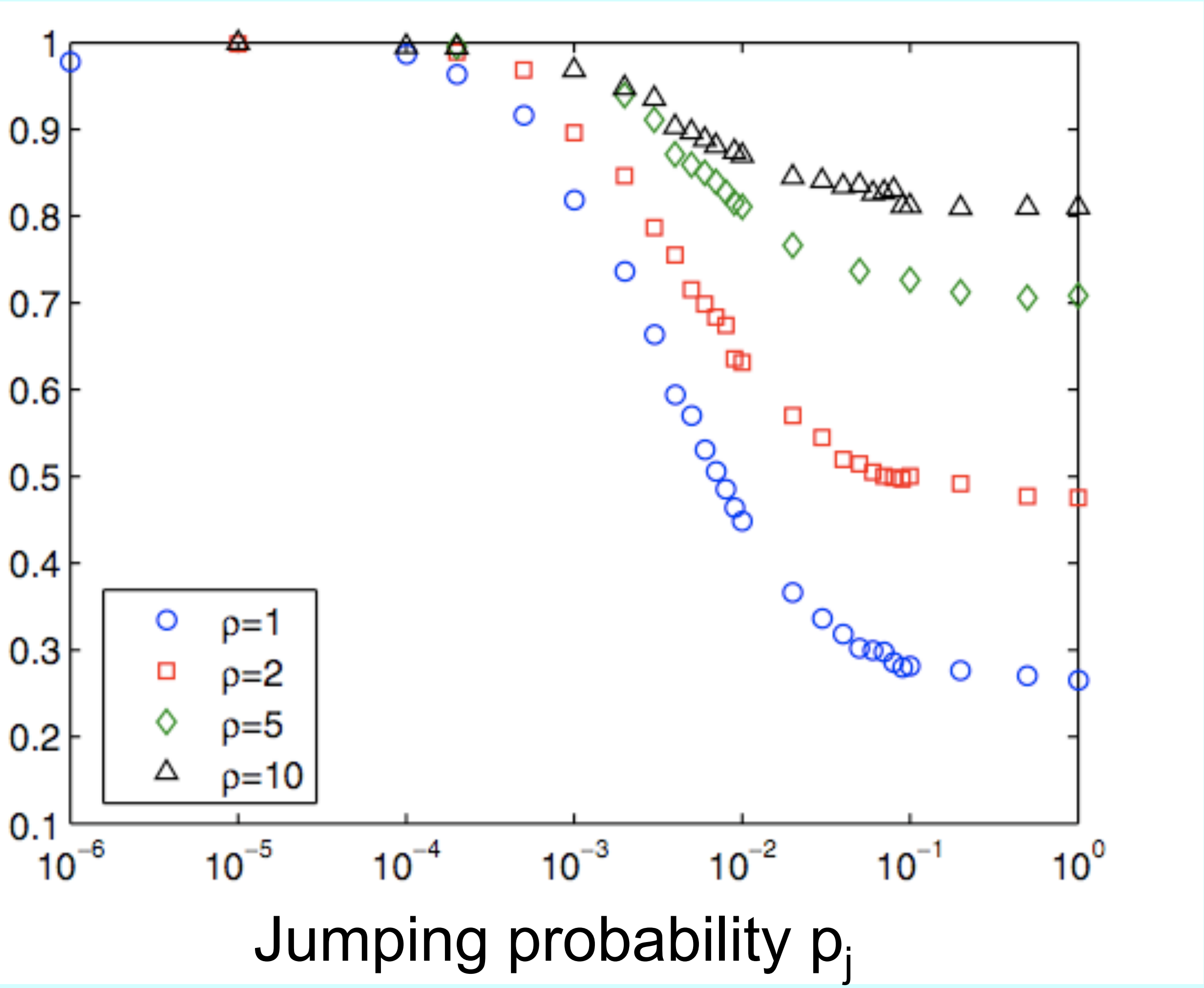


Jumping probability $p_j = 0.1$

Epidemic spreading in temporal small worlds

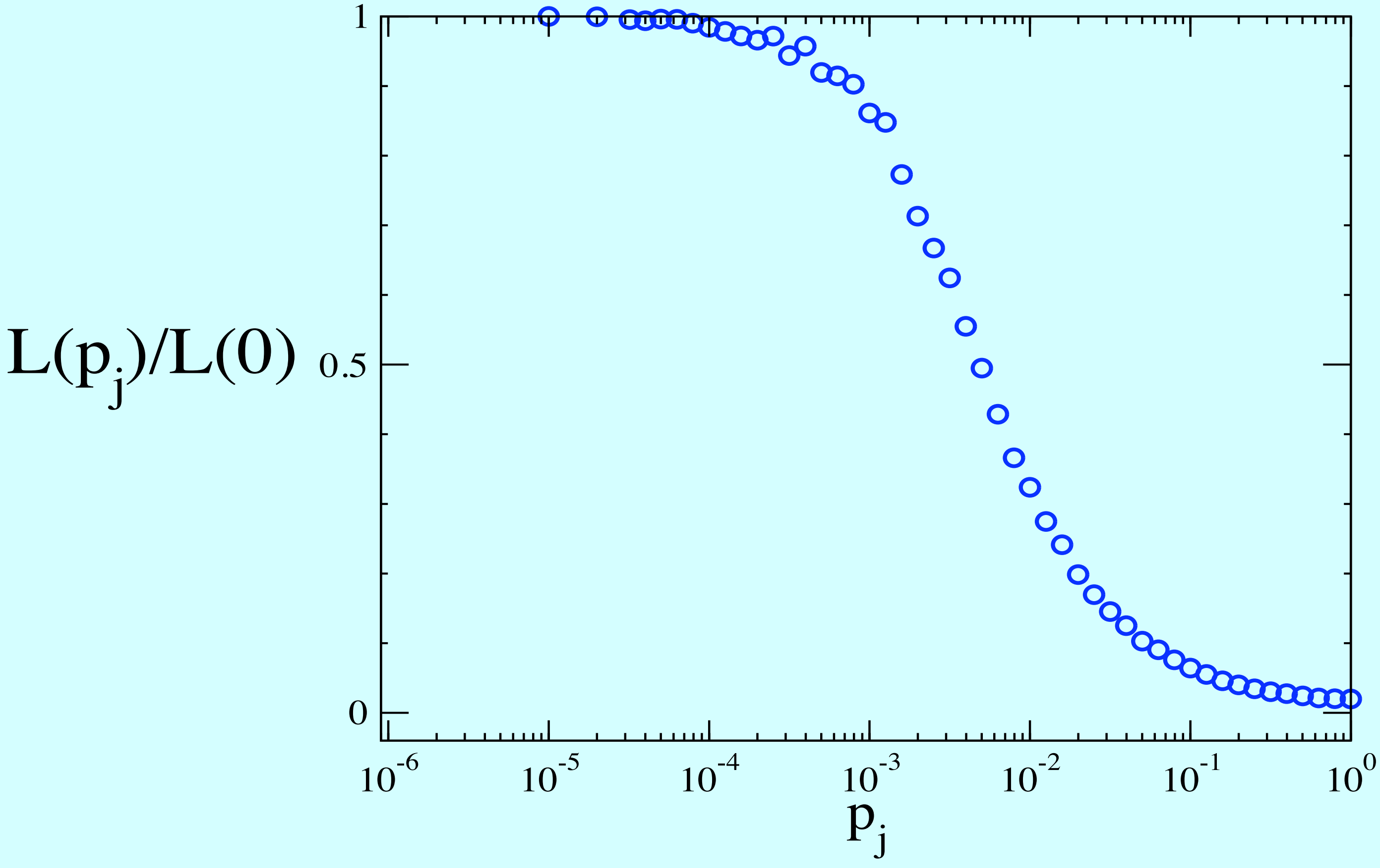


Critical threshold
$\lambda_c(p_j)$



Buscarino, Fortuna, Frasca, Latora EPL 82, 38002 (2008)

Epidemic spreading in temporal small worlds



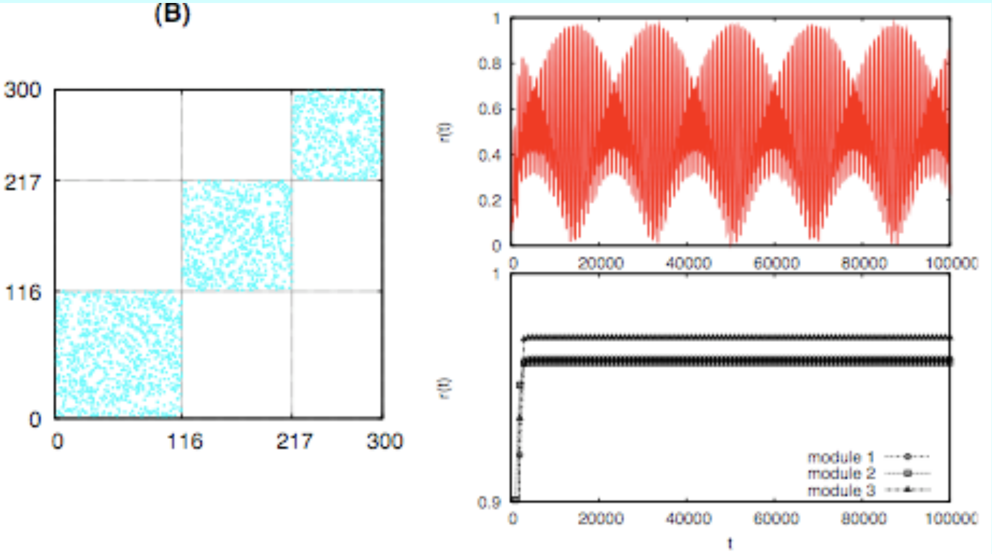
The epidemic threshold λ_c exhibits the same behavior as the average temporal distance L !!

Synchronization in time-varying networks

The nodes of the network are Kuramoto oscillators

1) HOMOPHILY: links between in-phase nodes are reinforced

2) COMPETITION: each node has a finite amount of resources to be distributed among the various links



Assenza et al, Scientific Reports 1, 99 (2011)
Gutierrrez et al, PRL 107, 234103 (2011)

Take-home message

- OLD: Many similar ideas in sociology, maths, and computer science literature

Cooke & Halsey, J. Math Analys Appl. (1966)……
……….
Kempe et al. (2002), Holme (2005), Pan & Saramaki (2011)

- NEW: Demands due to the increasing availability of large databases

The first review paper on "Temporal Networks", Holme, Saramaki, ArXiv 1108.1780 (2011)

- FUTURE ? Use also geography

Networks with detailed information both on space and on time:
FourSquare, Gowalla and other online location-based social networks

Collaborators

Nicosia, Sinatra, Assenza (Dept.Phys. CT)
Buscarino, Fortuna, Frasca, Russo (Electric.Engin CT)

Mascolo, Musolesi, Scellato,Tang (Comp. Lab. Cambridge)

Gomez-Gardenes (Zaragoza)

Boccaletti, Gutierrez (Madrid)

NEUROIMAGING LABS*: Babiloni* (Rome)
Chavez (Paris)

Synchronization in time-varying networks

$$\dot{\theta}_i = \omega_i + \lambda \sum_{j=1}^{N} W_{ij} \sin(\theta_j - \theta_i)$$

a net of Kuramoto oscillators

HOMOPHILY: links between in-phase nodes are reinforced

$$\dot{W}_{ij}(t) = W_{ij}(t)\left[s_i \cdot p_{ij}^T(t) - \sum_{l=1}^{N} W_{il}(t) \cdot p_{il}^T(t)\right]$$

$$p_{ij}^T(t) = \left|\frac{1}{T}\int_{t-T}^{t} e^{i[\theta_j(\tau)-\theta_i(\tau)]} d\tau\right|$$

$$s_i = \sum_{j=1}^{N} W_{ij}$$

Total in-strength

COMPETITION: each node has a finite amount of resources

$$\dot{s}_i = \sum_{j=1}^{N} \dot{W}_{ij} = 0$$

Assenza et al, Scientific Reports 1, 99 (2011)
Gutierrrez et al, PRL 107, 234103 (2011)