Networks Through a Quantum Lens

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University College London

Cambridge Networks Day 2012

Preface

"Our children no longer want to become physicists and astronauts.

They want to invent the next Facebook instead.

They don't talk quanta — they dream bits.

They don't see entanglement but recognize with ease nodes and links."

A.-L. Barabási, *Nature Physics* (2011)

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Quantum Information studies how to encode, process and transfer bits stored in quantum systems (atoms, photons, *etc.*)

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Quantum Information studies entanglement and sometimes it is about nodes and links

Introduction

Quantum Information (QI)

Features & Merits:

1. Information is tied to a physical representation
2. The quantum world can be engineered
3. Interesting conceptual & mathematical toolbox
4. *App.*: QI guarantees secure communication
5. *App.*: QI clarifies the limits of computation

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Issues:

1. Quantum vs. Gravity
2. The macroscopic world is classical
3. Implementability of protocols

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1. What is the interface with networks?
2. Can we use the toolbox of QI in Network Theory?

Issues:

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2. The macroscopic world is classical
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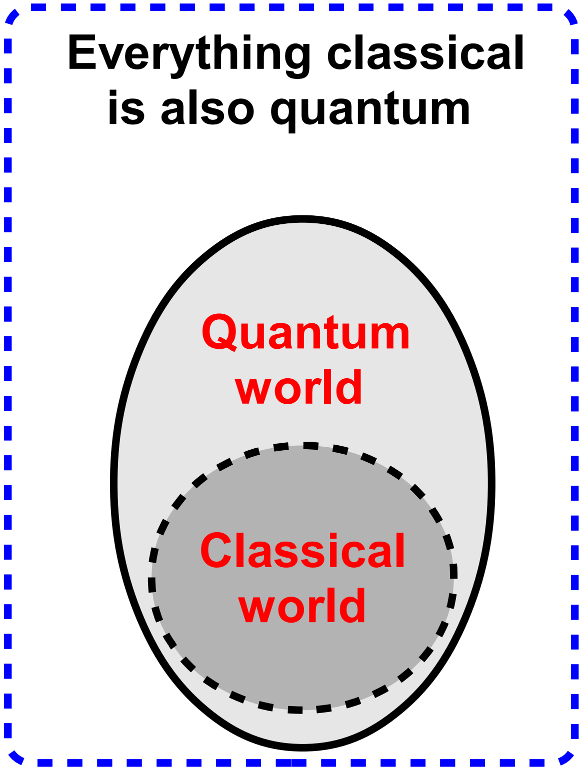
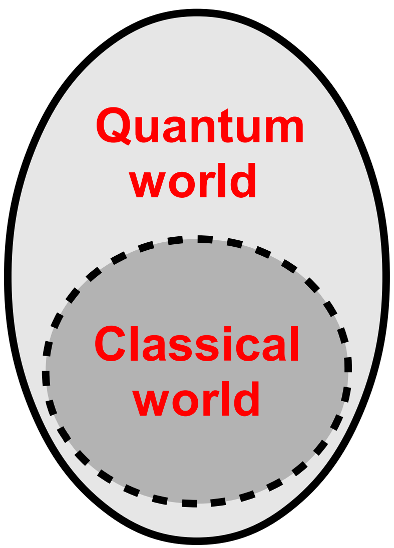
Quantum features

Quantum features



Quantum features

Everything classical is also quantum



Mathematical axioms

Physical states are matrices generalizing probability distributions

Composite systems need a tensor product

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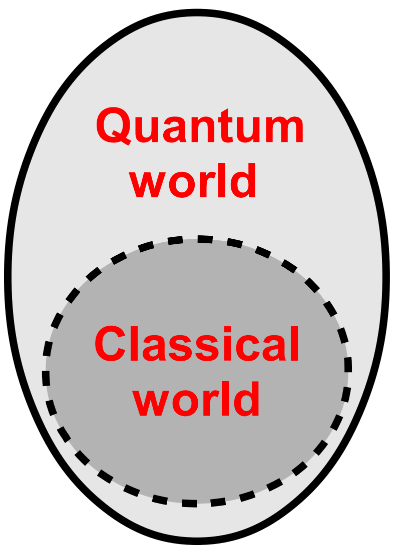

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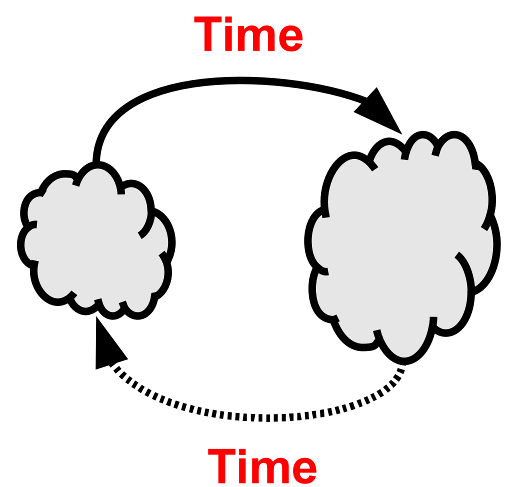

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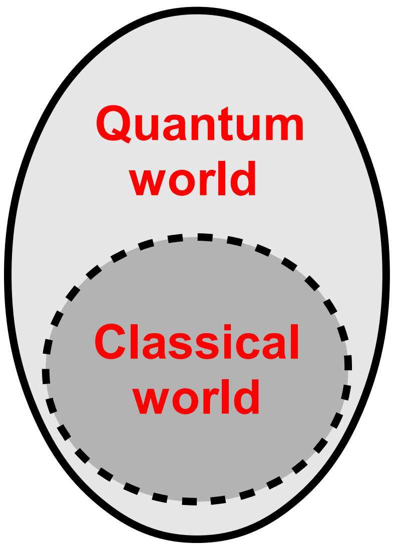

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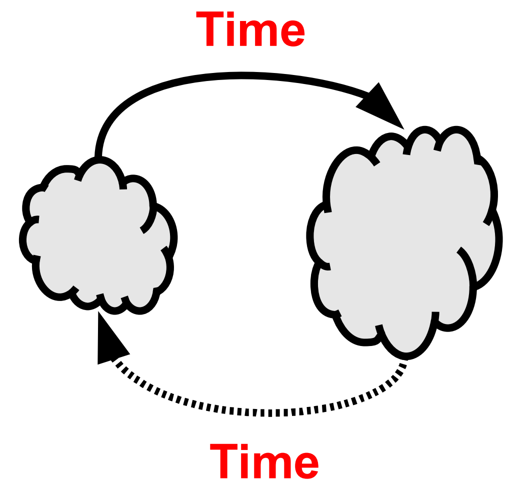
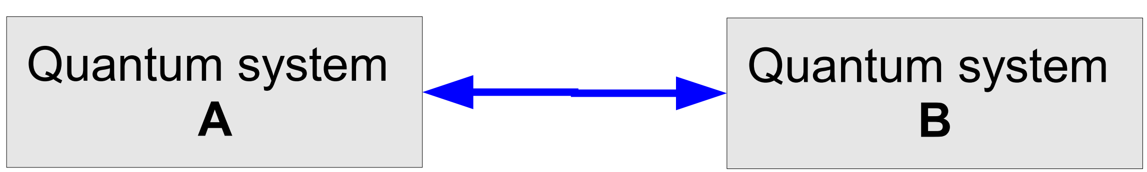
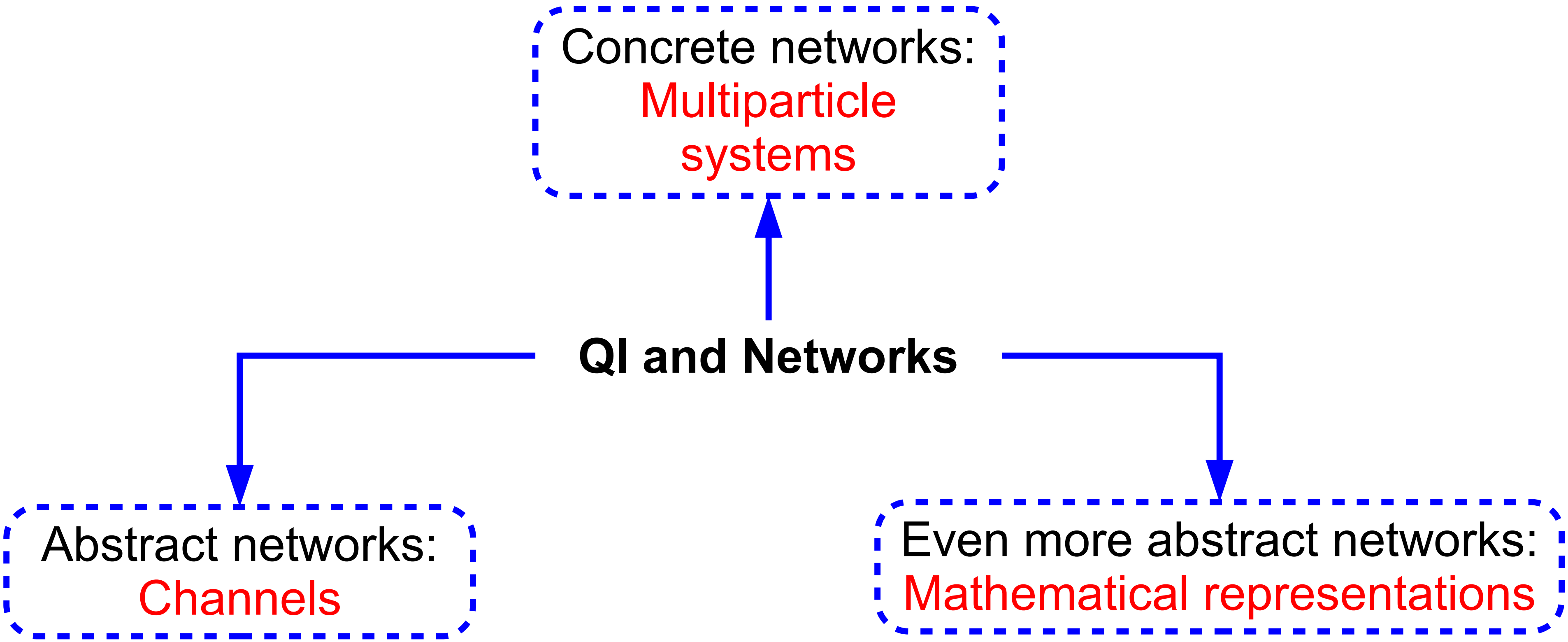


Stronger than classical correlations

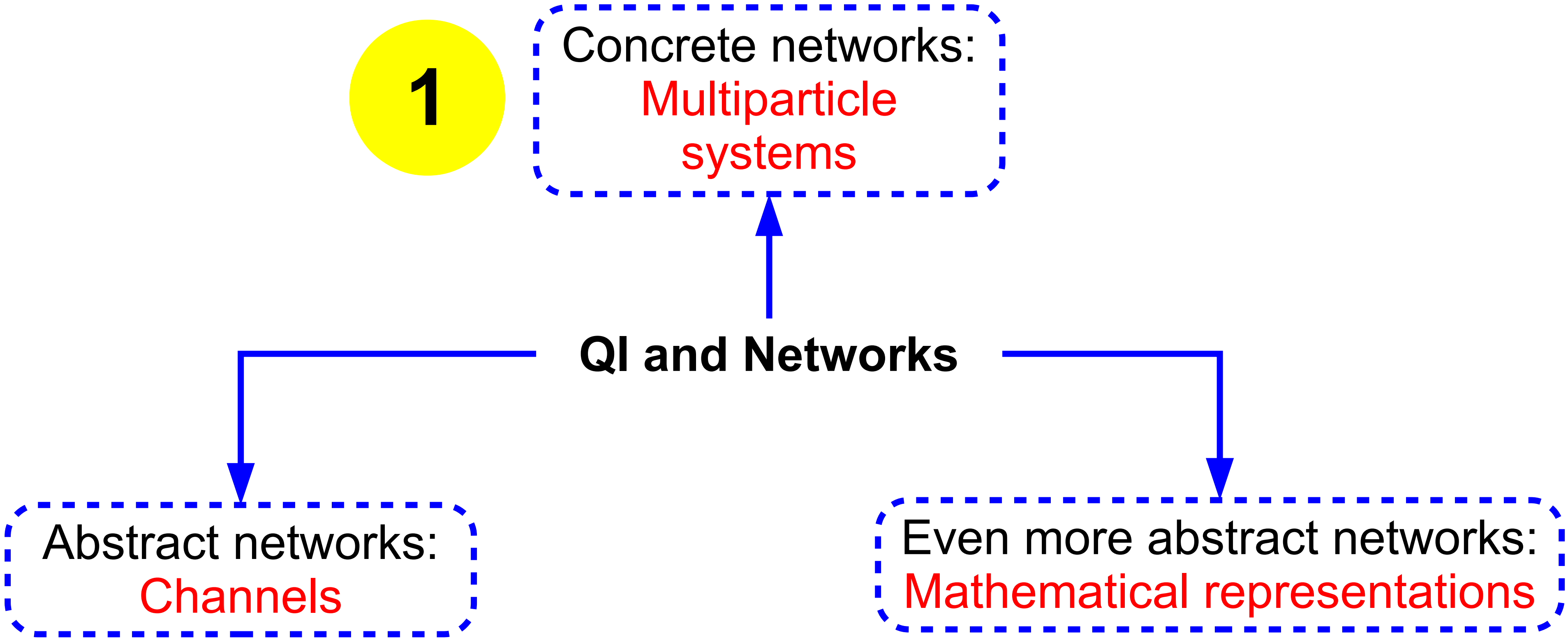


Plan



Concrete networks:
Multiparticle systems
QI and Networks
Abstract networks:
Channels
Even more abstract networks:
Mathematical representations

Plan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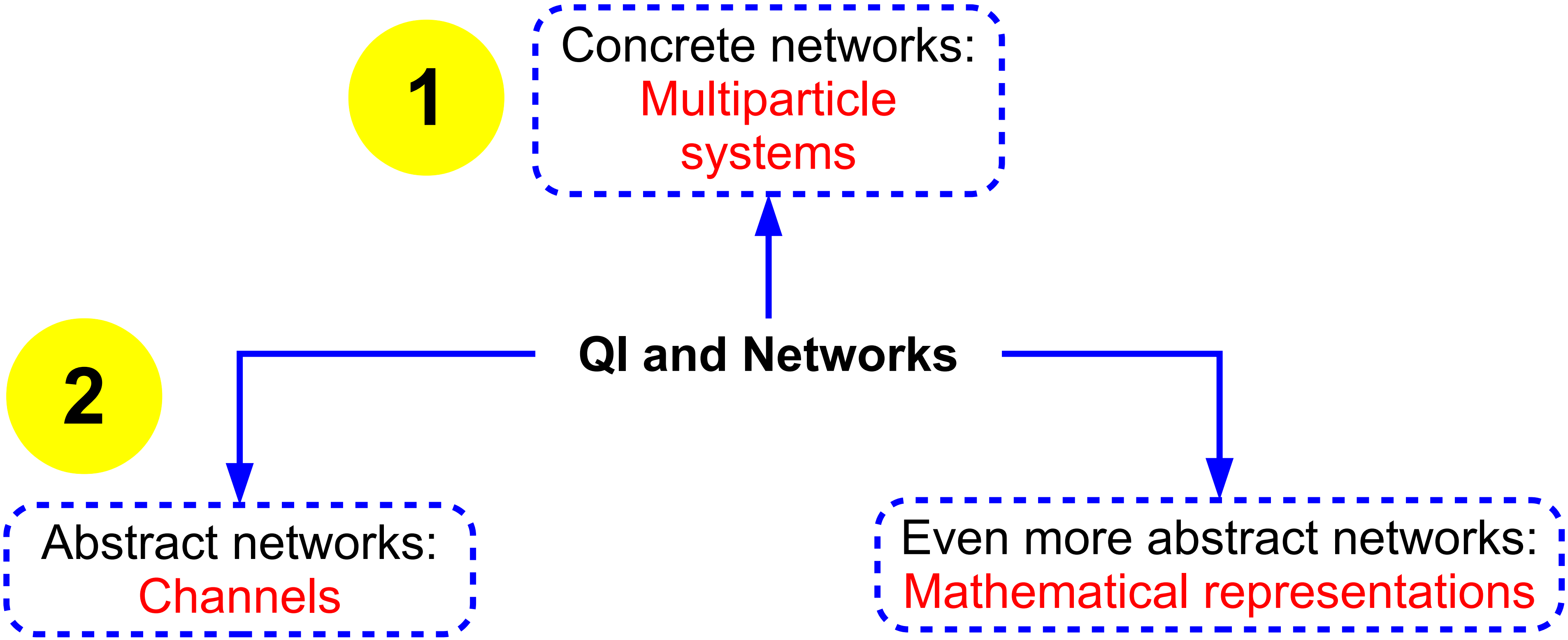
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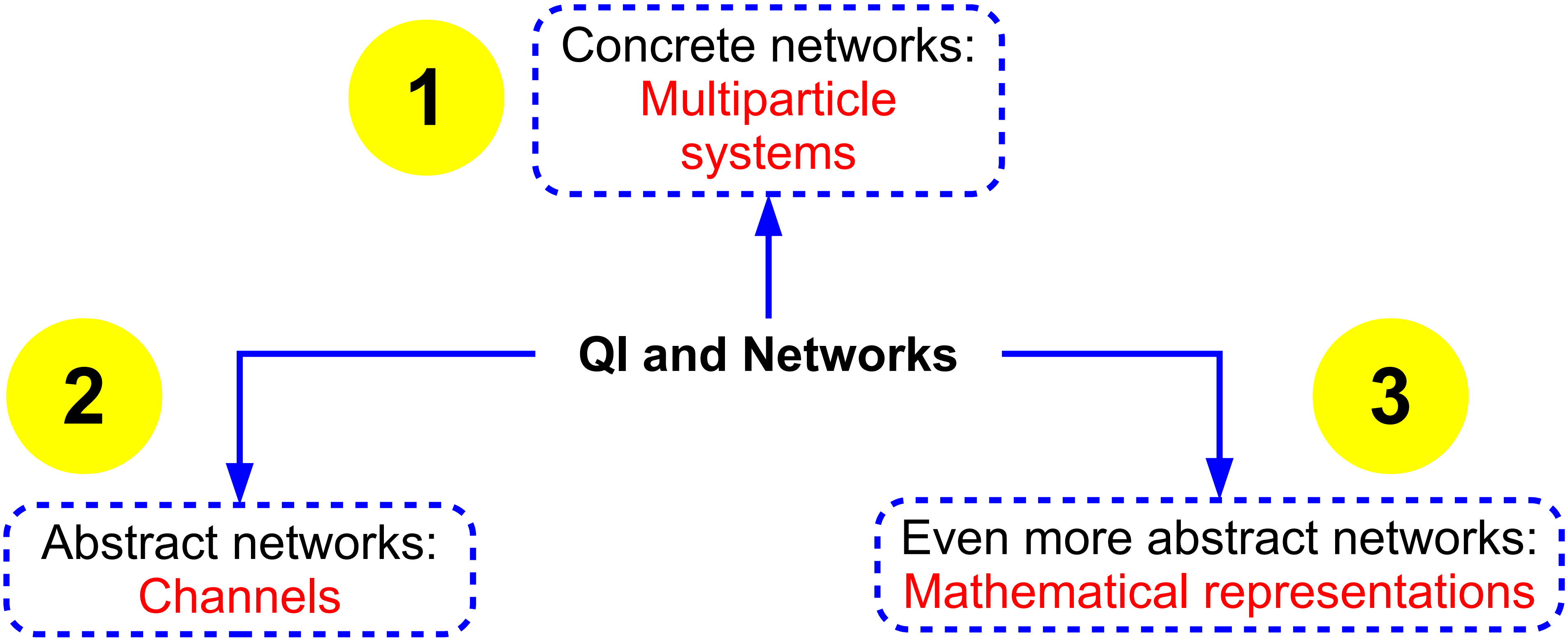
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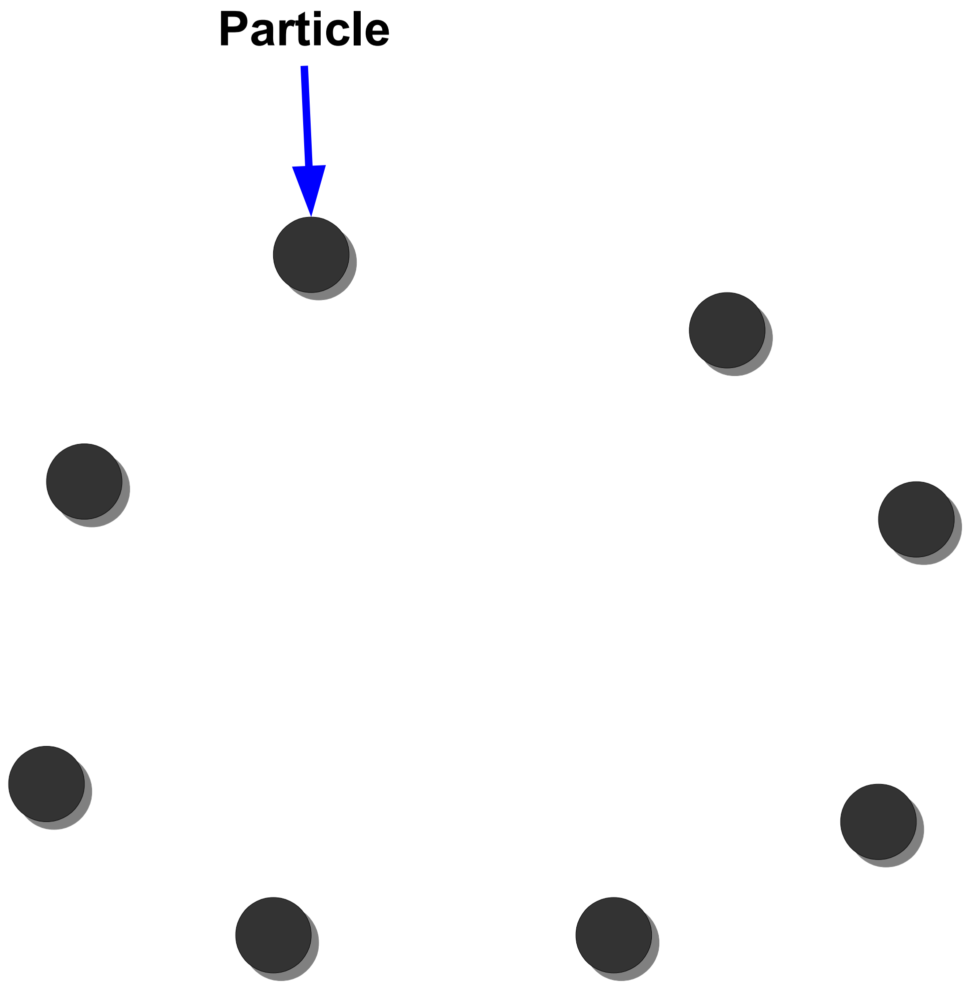
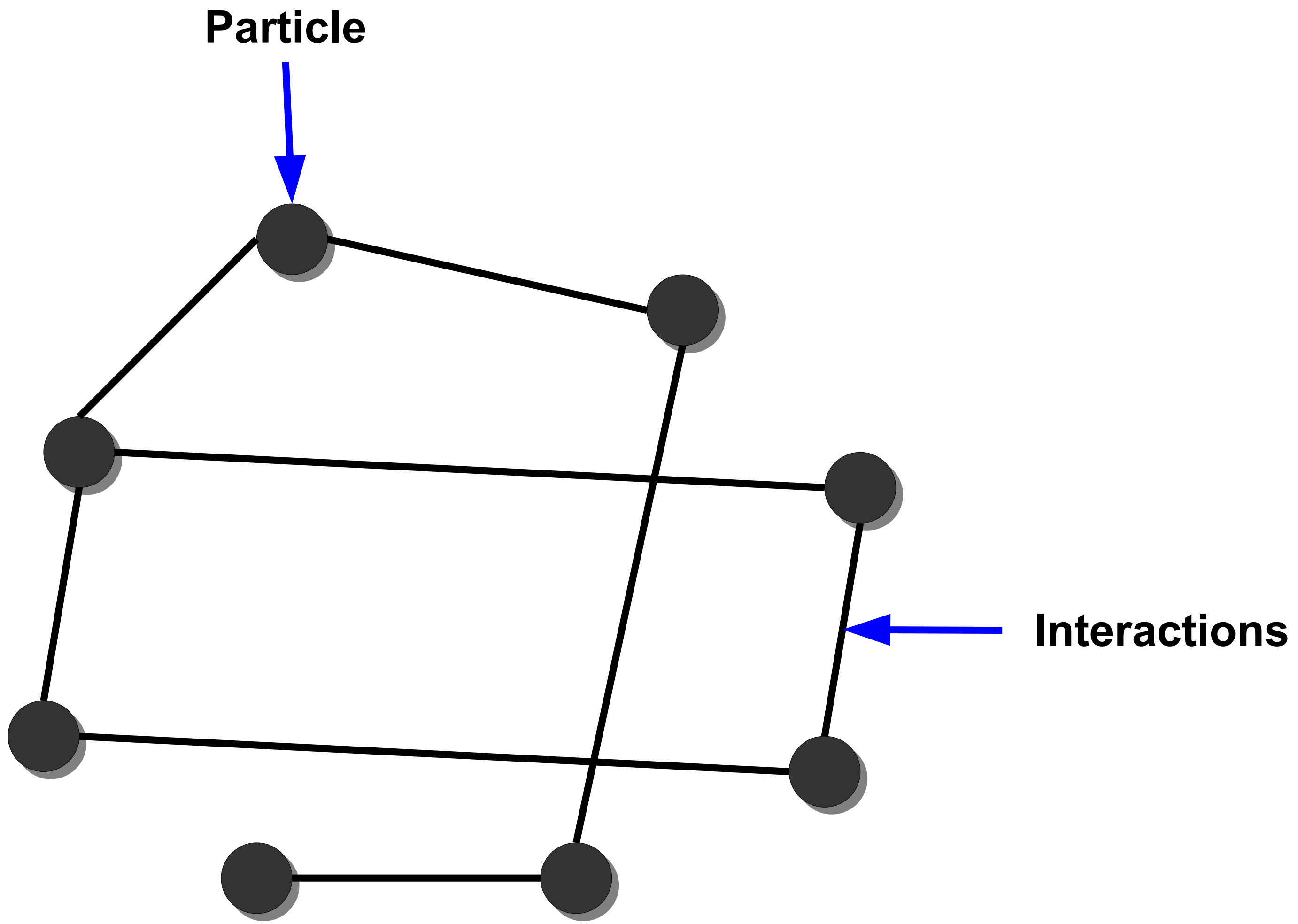
1

Concrete networks

Multiparticle systems



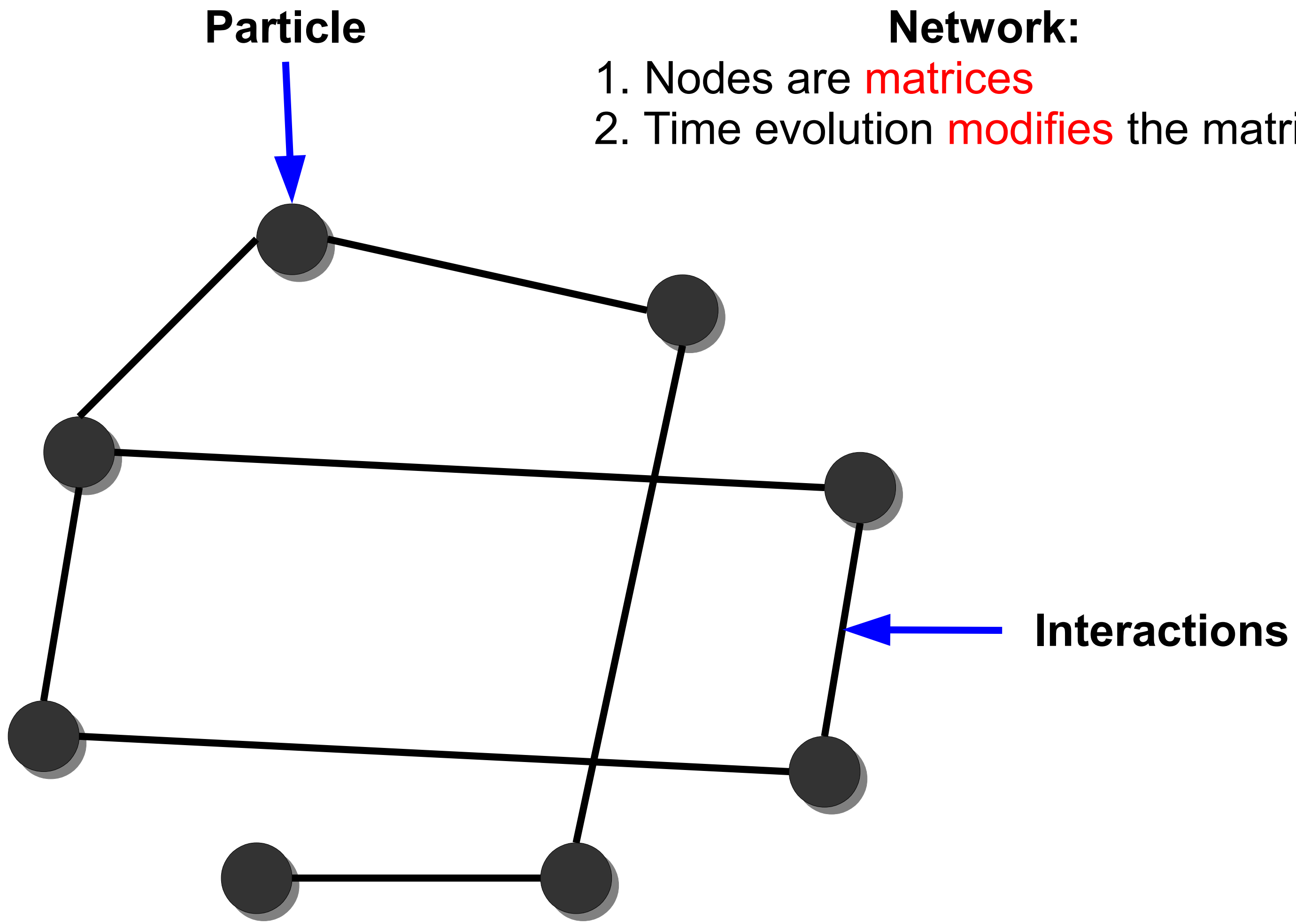
Multiparticle systems



Multiparticle systems

Network:

1. Nodes are matrices
2. Time evolution modifies the matrices



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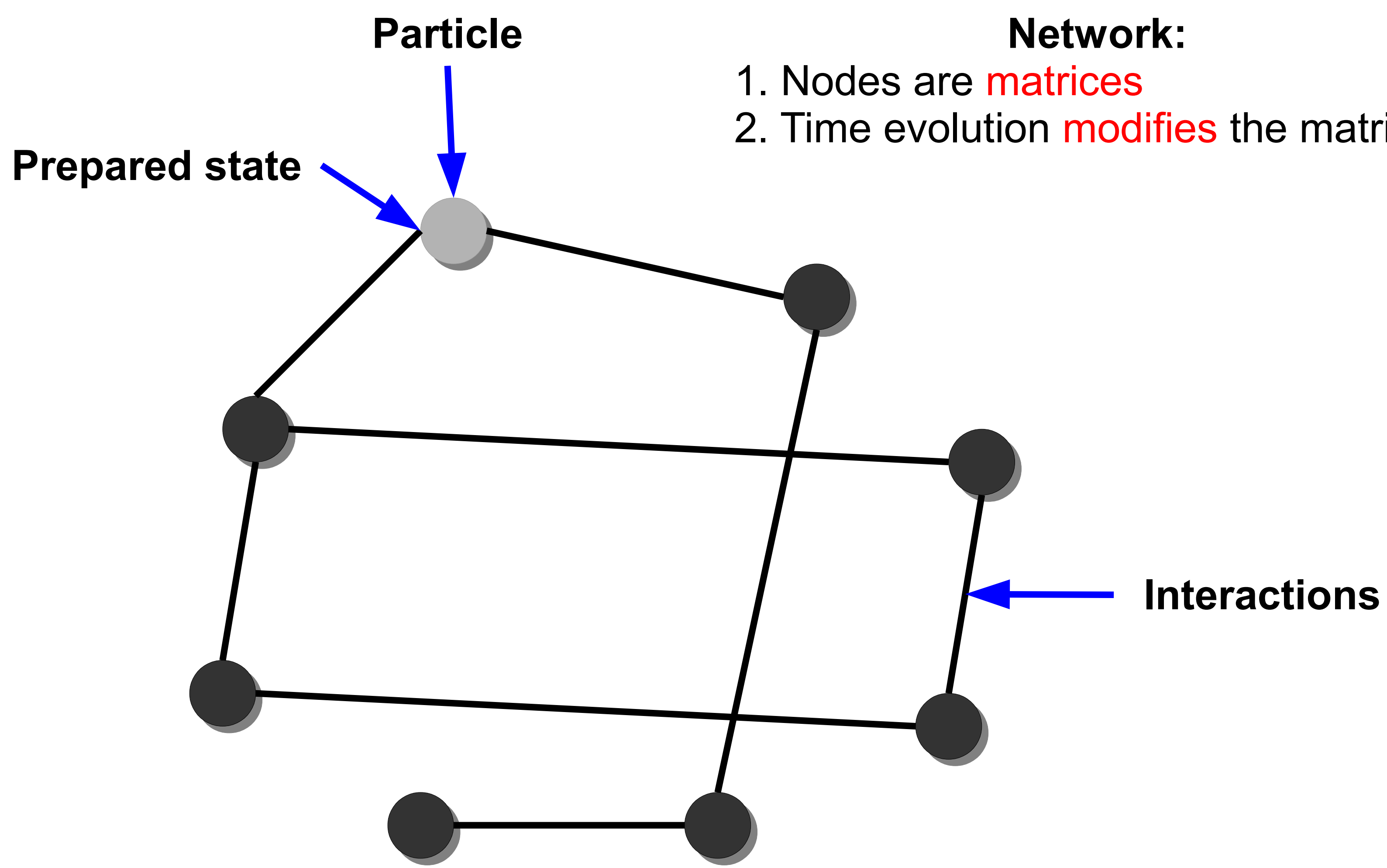

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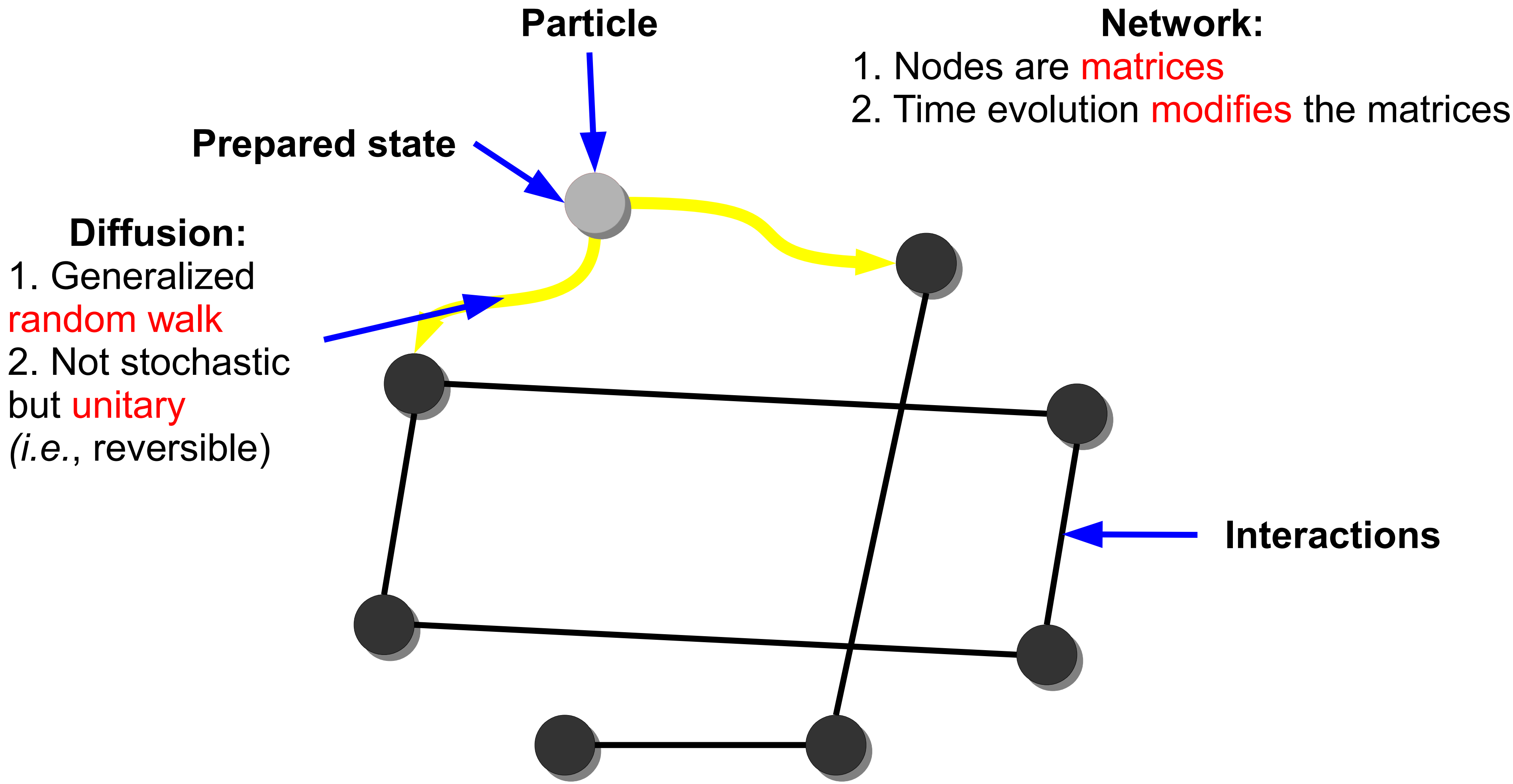
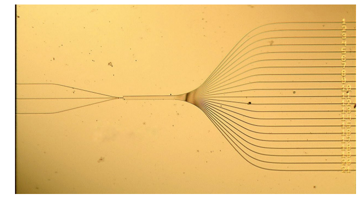
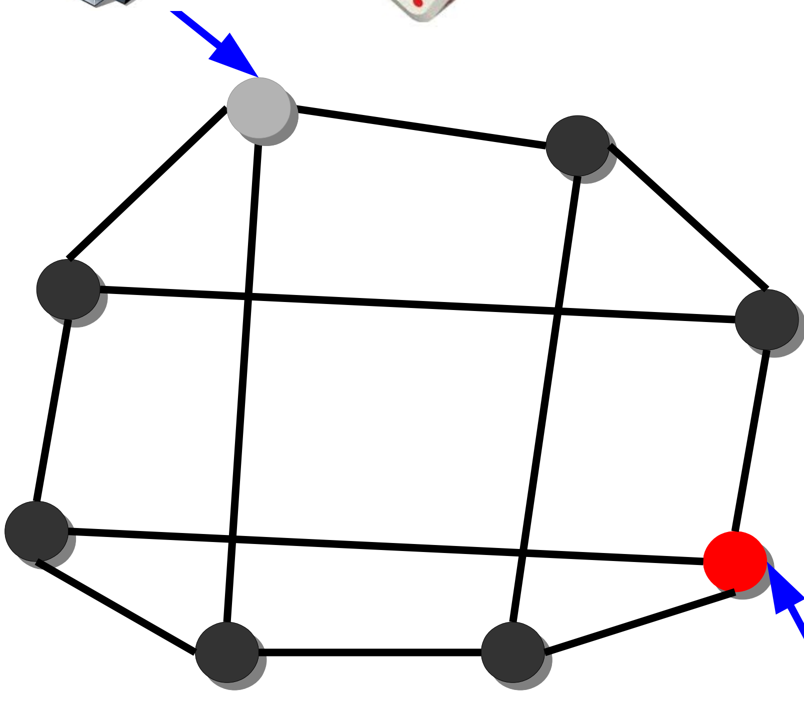


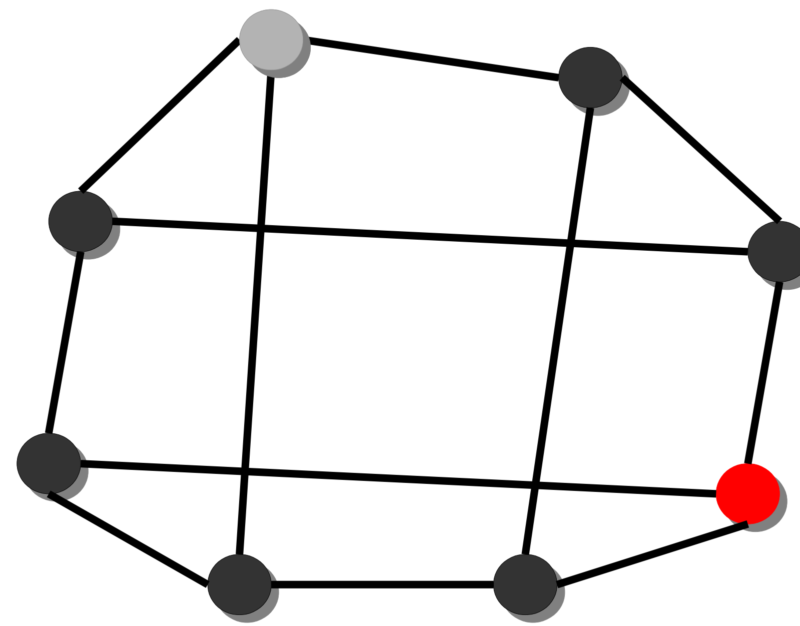
State transfer

Dice

Optical waveguides







Random walk

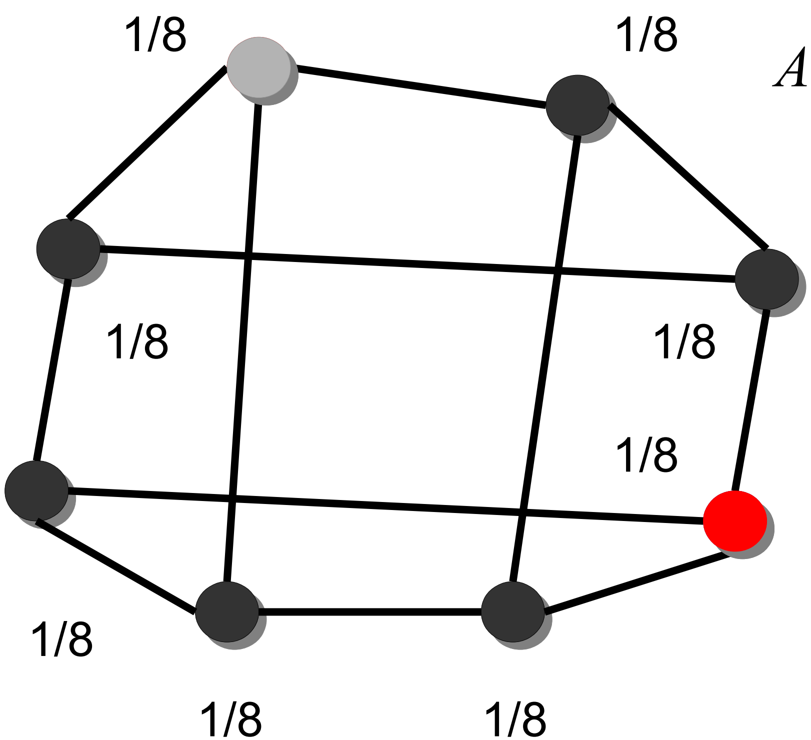
Quantum walk

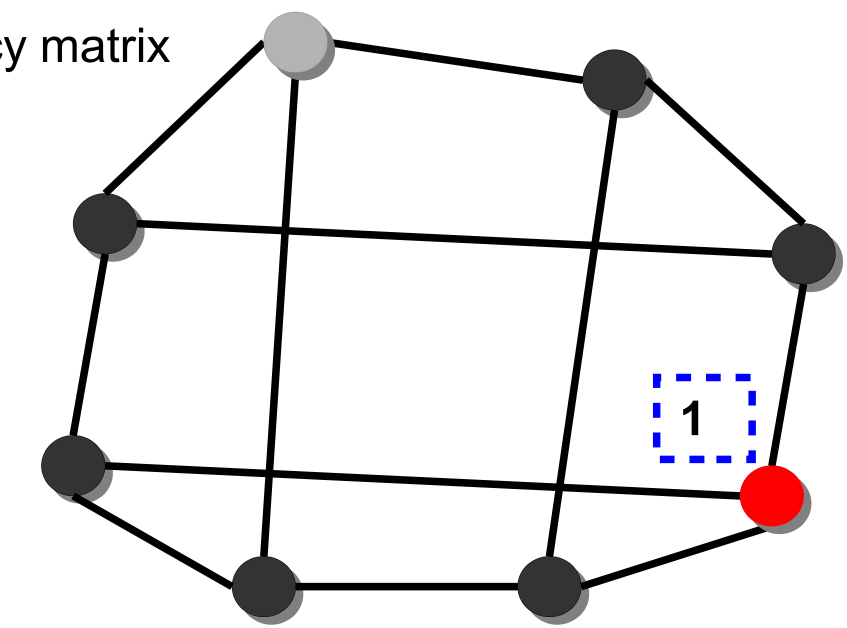
State transfer

$A^t * v_0 = v_t$

$\exp[A * i * t] v_0 = v_t$

A is the adjacency matrix



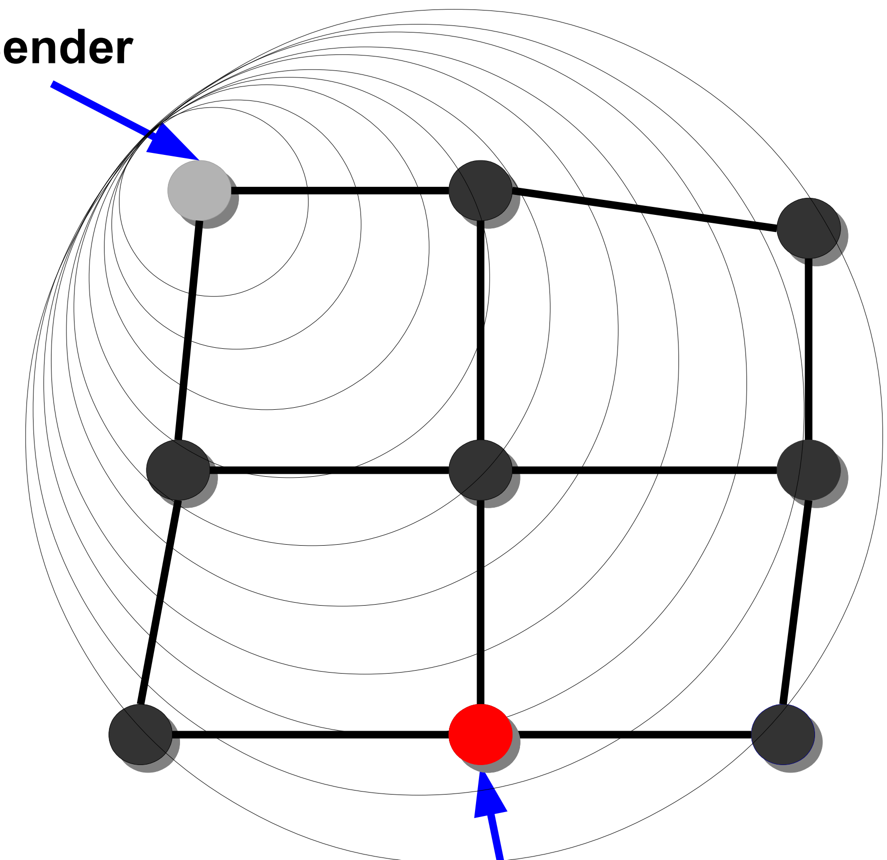


Random walk

Quantum walk

S. Bose, *Phys. Rev. Lett.* **91**, 207901 (2003)

Wave communication

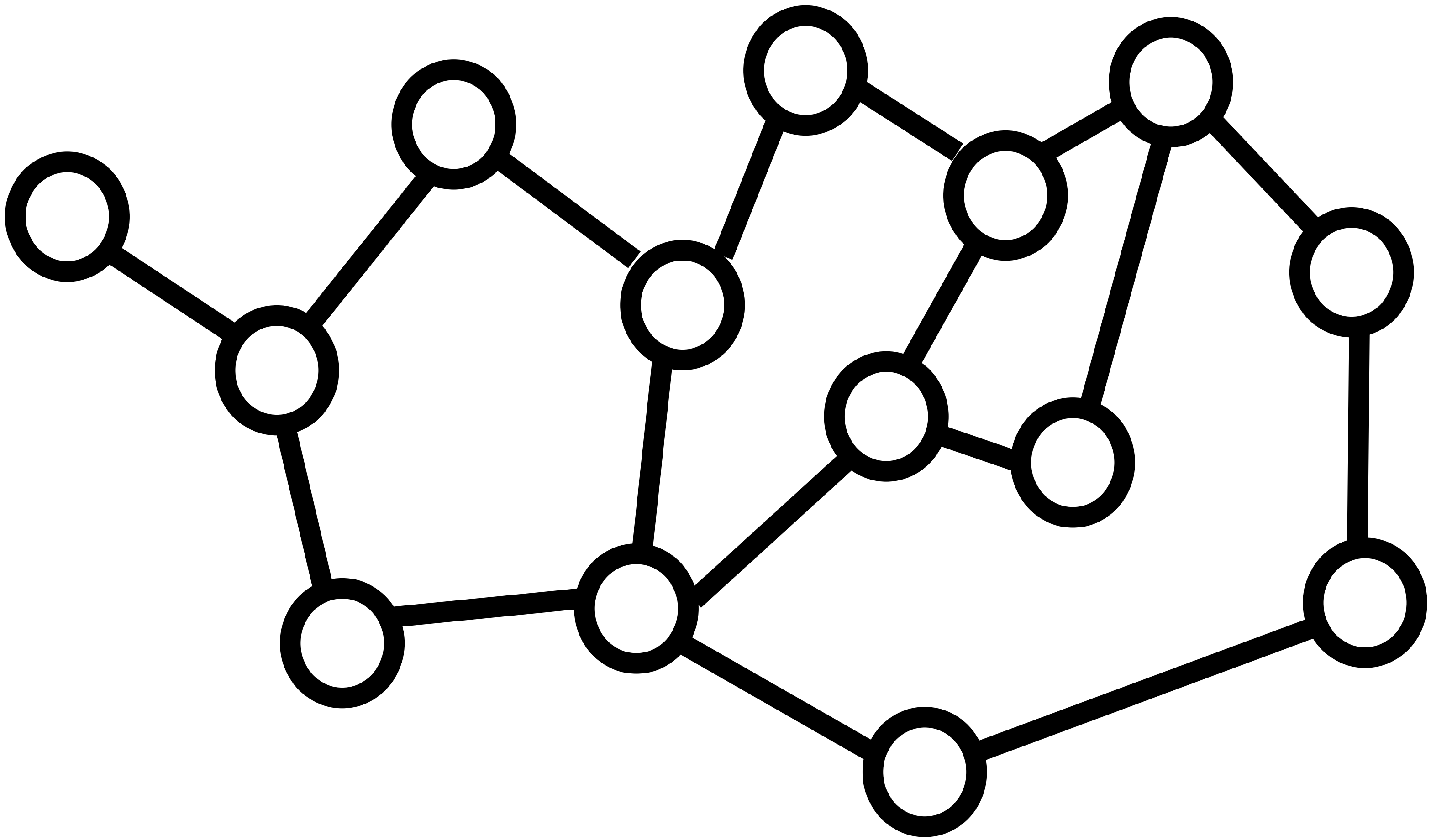


The signal explores the network but at a given time it is only at the target node

(Why? Interference effects depending on the structure of the network)

B. Hein, G. Tanner, *Phys. Rev. Lett.* **103**, 260501 (2009)

Control



Goal: transform the whole system in a desired state
Task: act on the minimum number of particles

D. Burgarth, V. Giovannetti, *Phys. Rev. Lett.* **99**, 100501, (2007)

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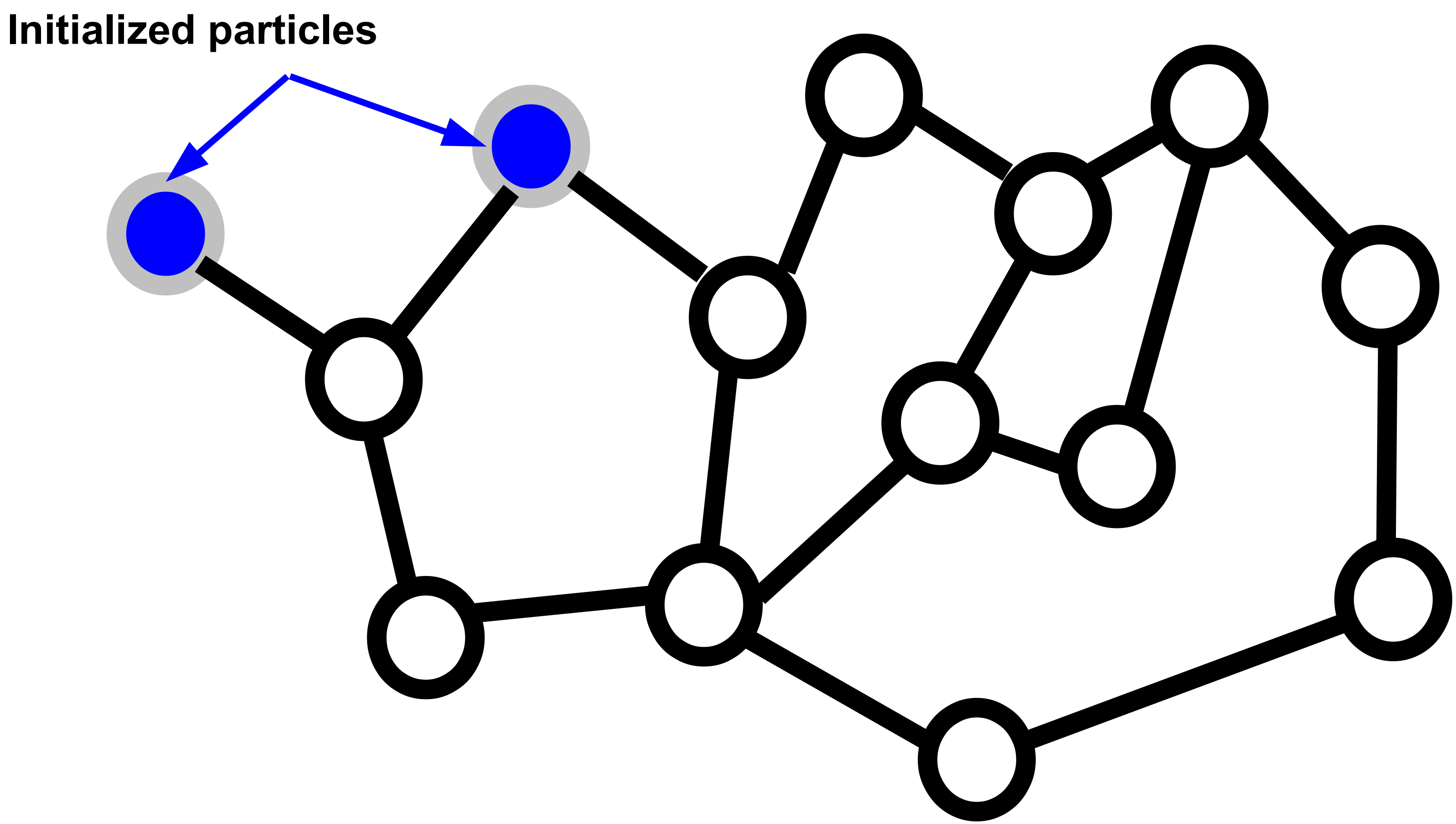

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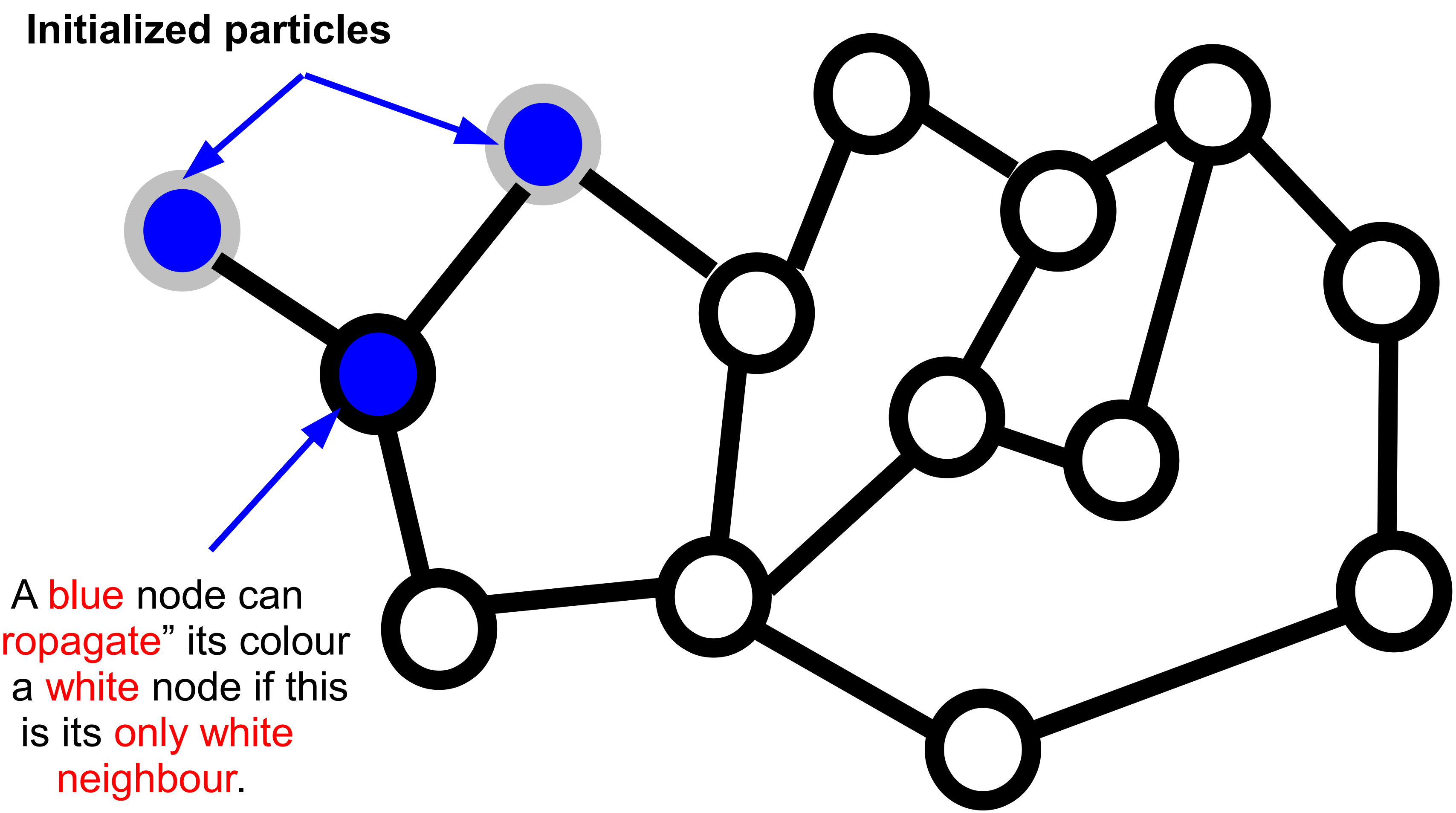

A blue node can "propagate" its colour to a white node if this is its only white neighbour.

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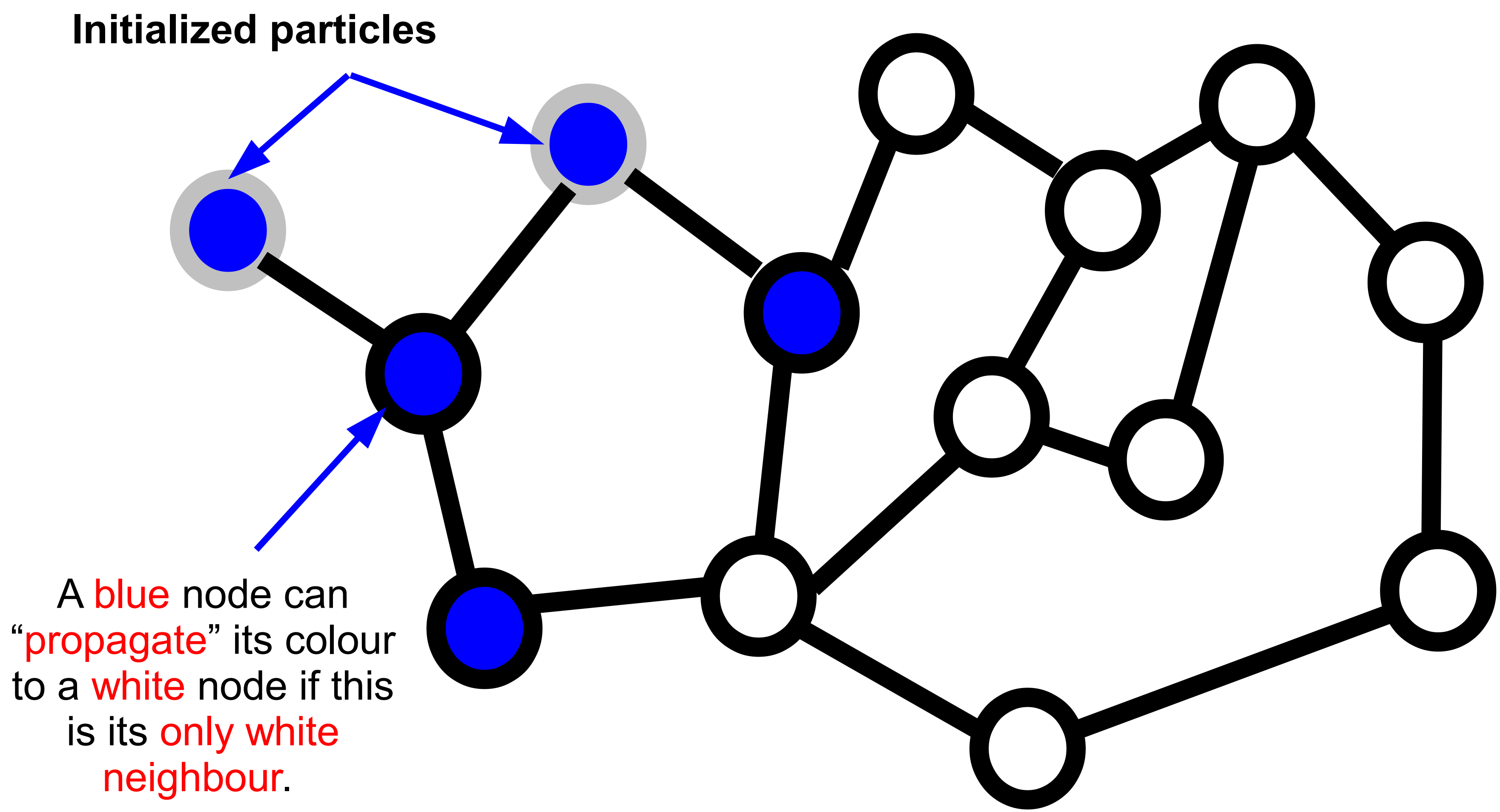

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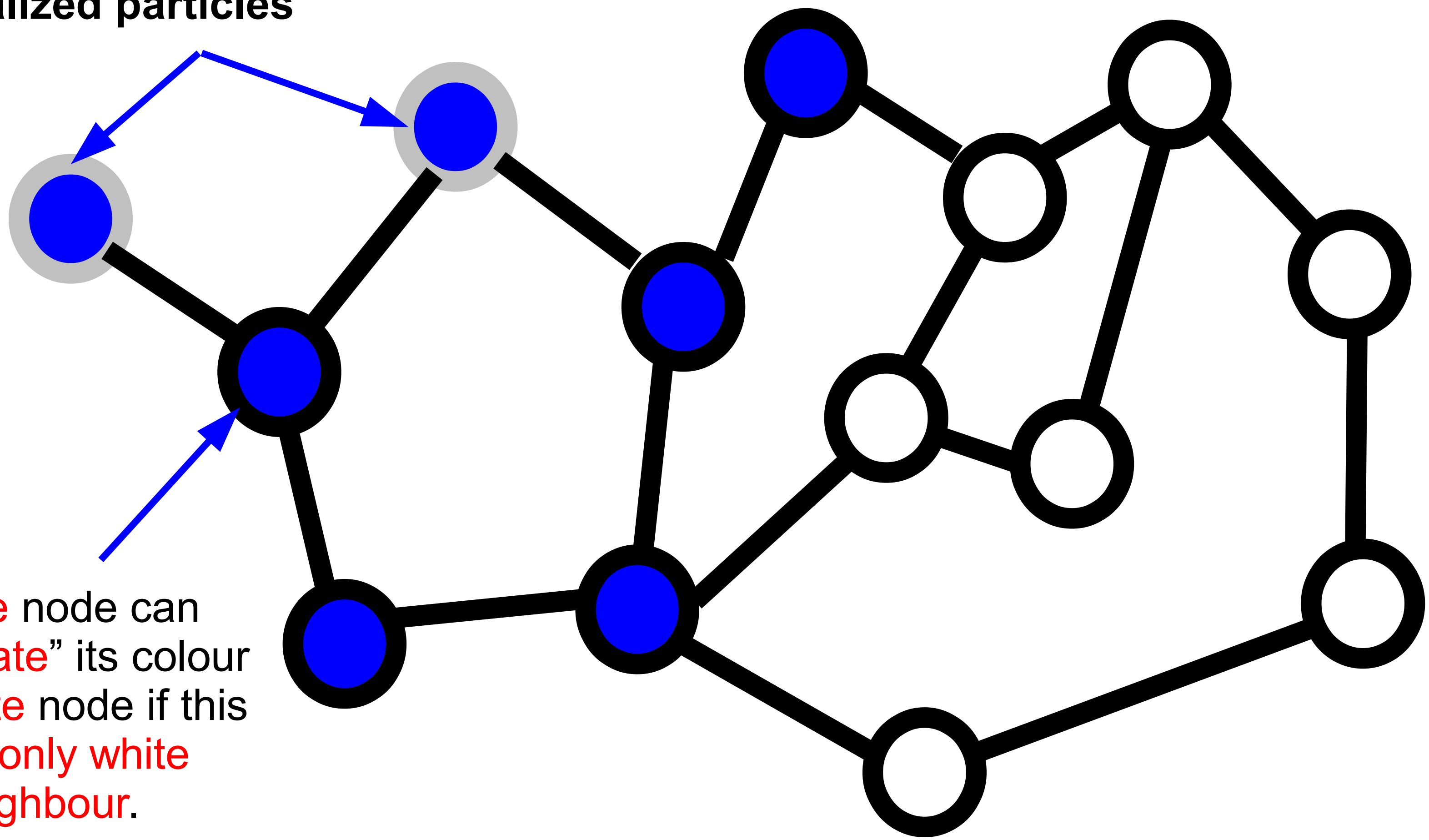
Initialized particles



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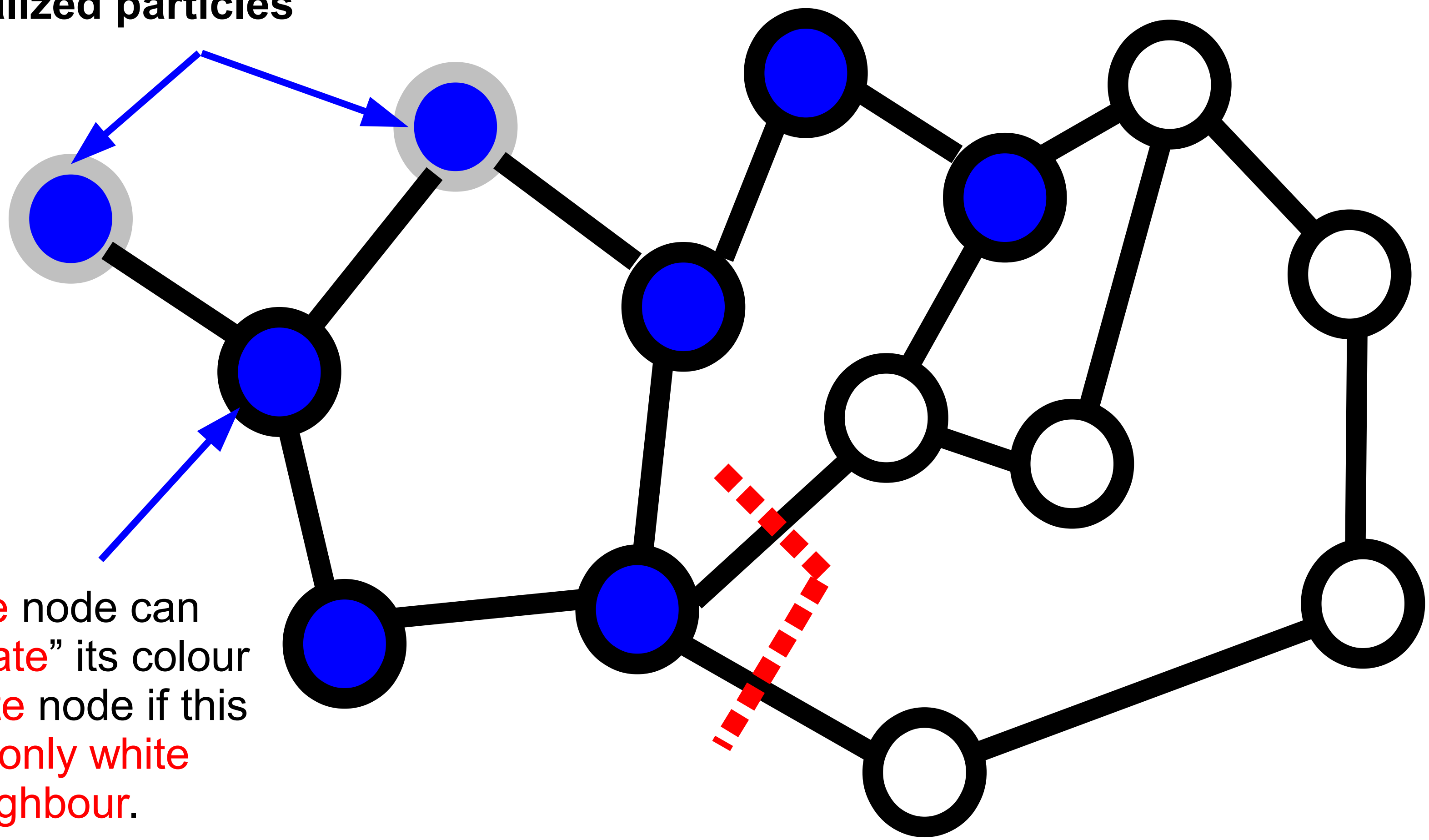

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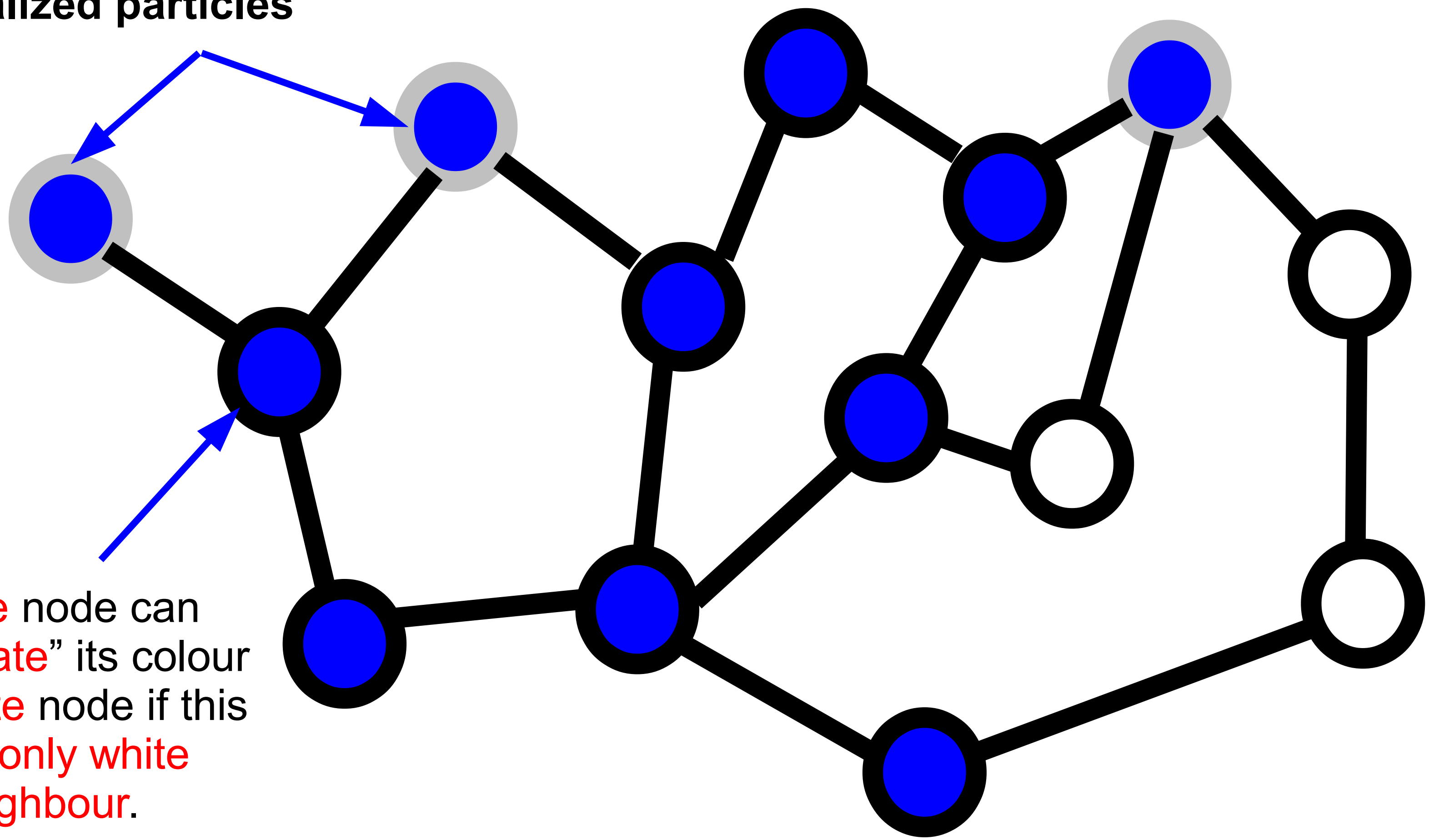

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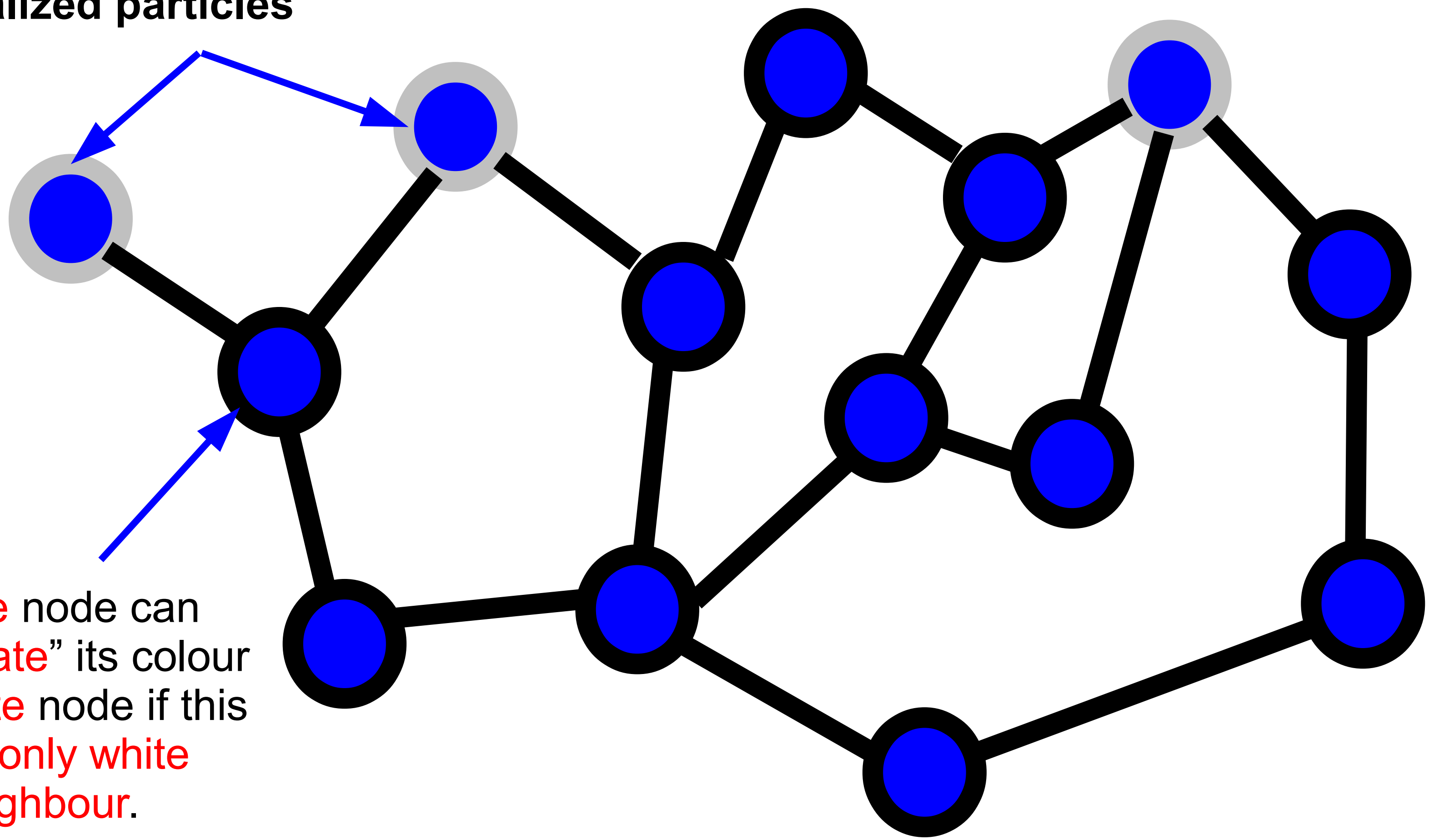

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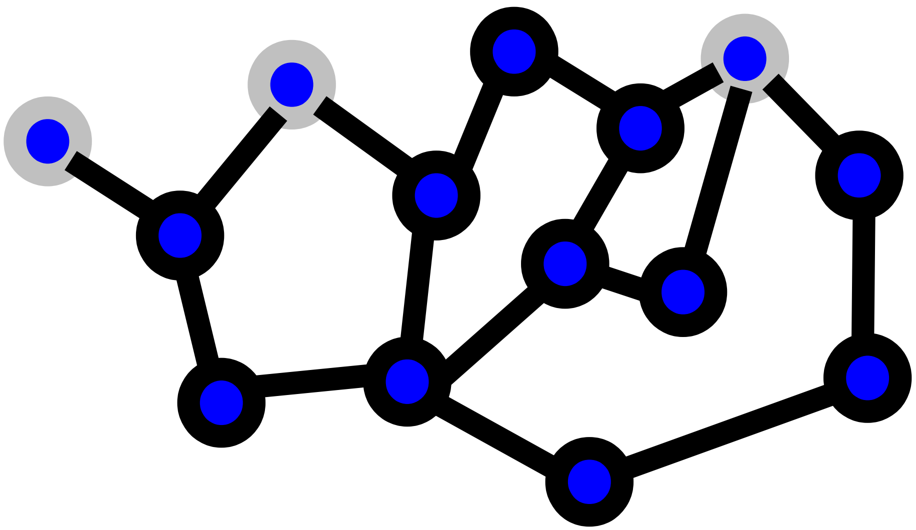
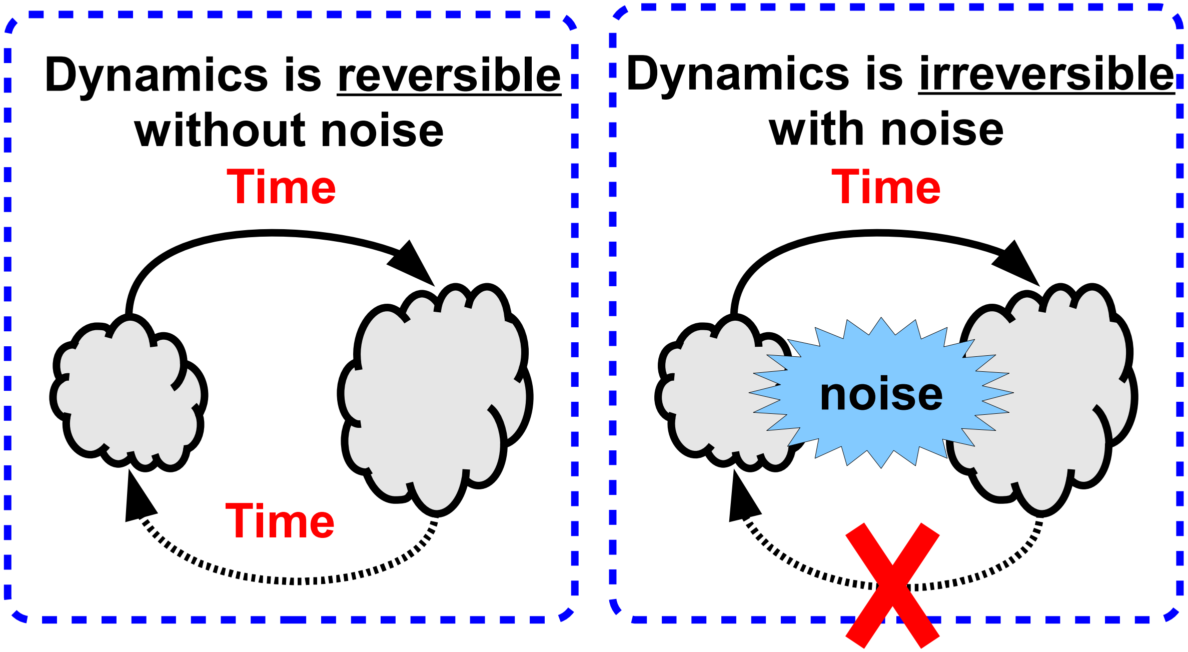


Punchline:
1. This game is related to Kalman's Rank Condition for controllability
2. NP-hard
3. Many variants

For the mathematical palate: the projection operator induced by the characteristic vector of the initially coloured vertices together with the adjacency matrix of the graph generate the set of all skew-symmetric matrices.

D. Burgarth, V. Giovannetti, *Phys. Rev. Lett.* **99**, 100501, (2007)

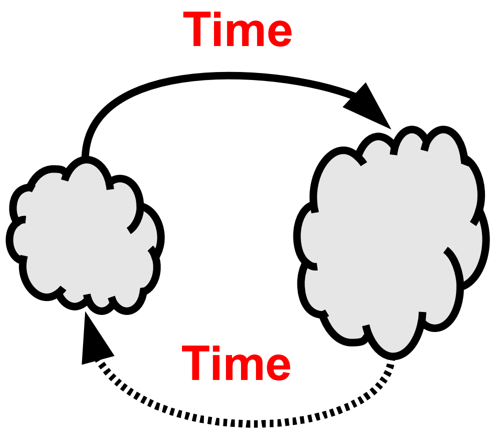
Noise



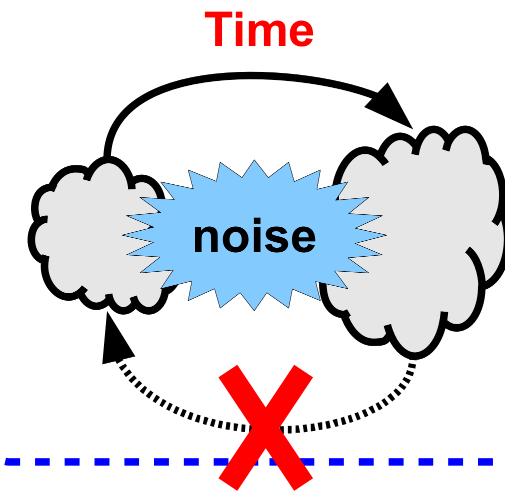
M. Mohseni, P. Rebentrost, S. Lloyd, A. Aspuru-Guzik,
J. Chem. Phys. **129**, 174106 (2008)

Noise

Dynamics is <u>reversible</u> without noise

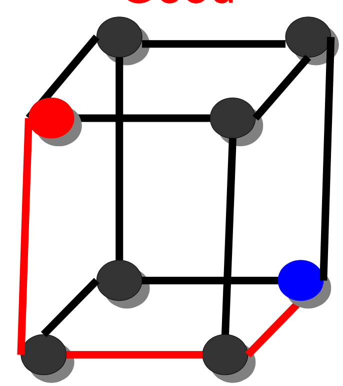


Dynamics is <u>irreversible</u> with noise



Without noise:

High symmetry: Good



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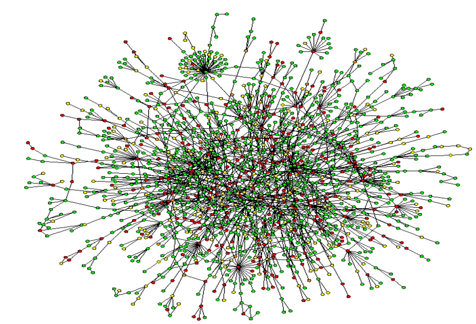

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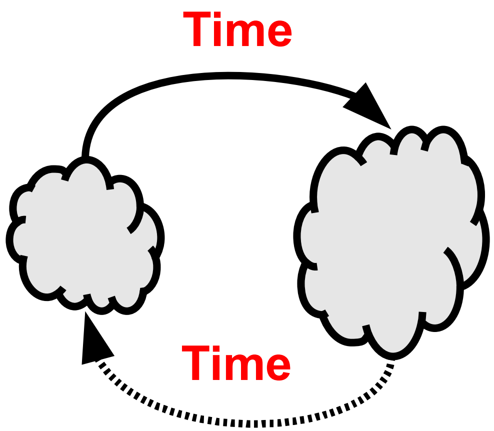

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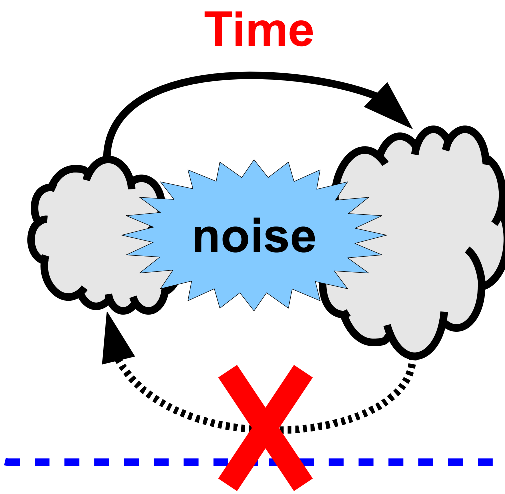

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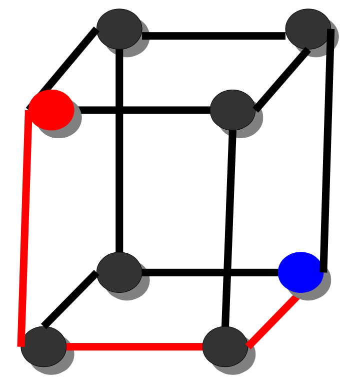

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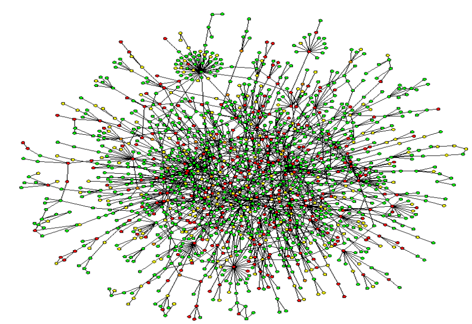

With noise:

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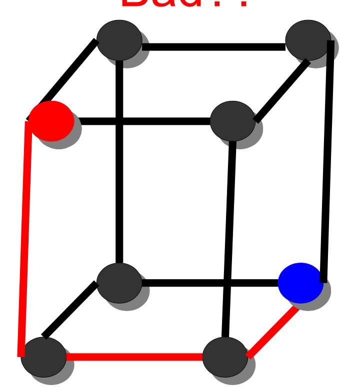

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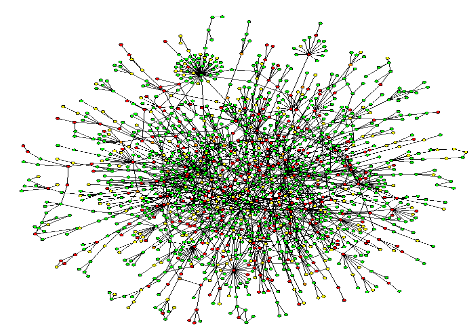

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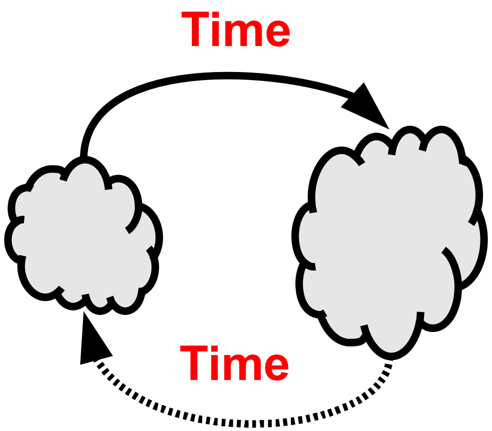

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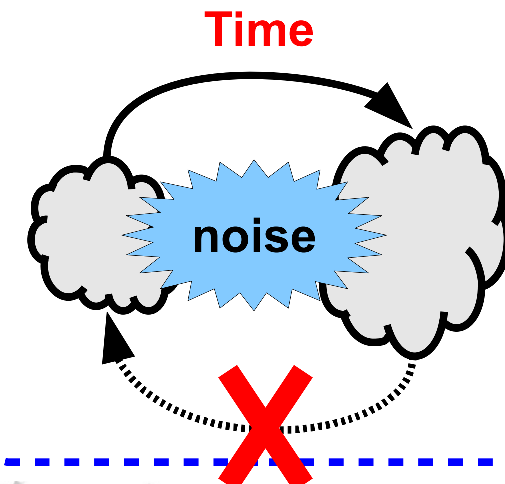

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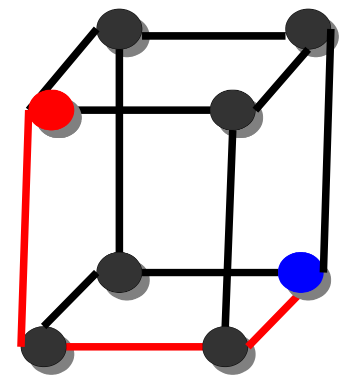

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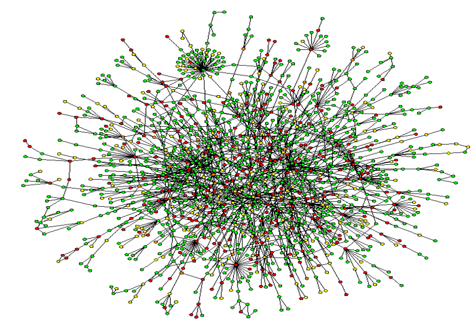
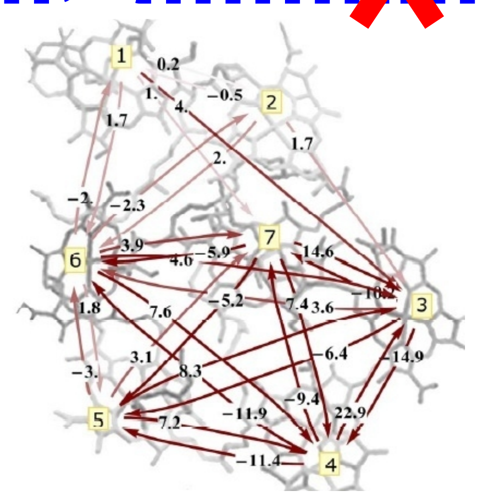


Multichromophoric energy transfer



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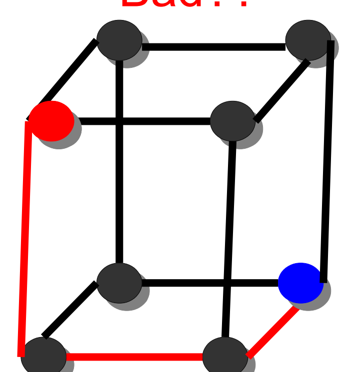

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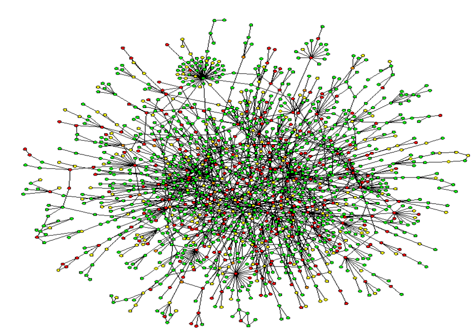


Summary 1

News:

1. A new paradigm for information transfer in networks
2. New methods to explore networks
3. New phenomena occurring on networks
4. New network-theoretic parameters
5. A way to encode information in nodes and links

Problems:

1. What is the role of network structure?
2. What about classifying networks according to their “quantum” properties?
3. What are the best networks for optimal energy/information transfer?

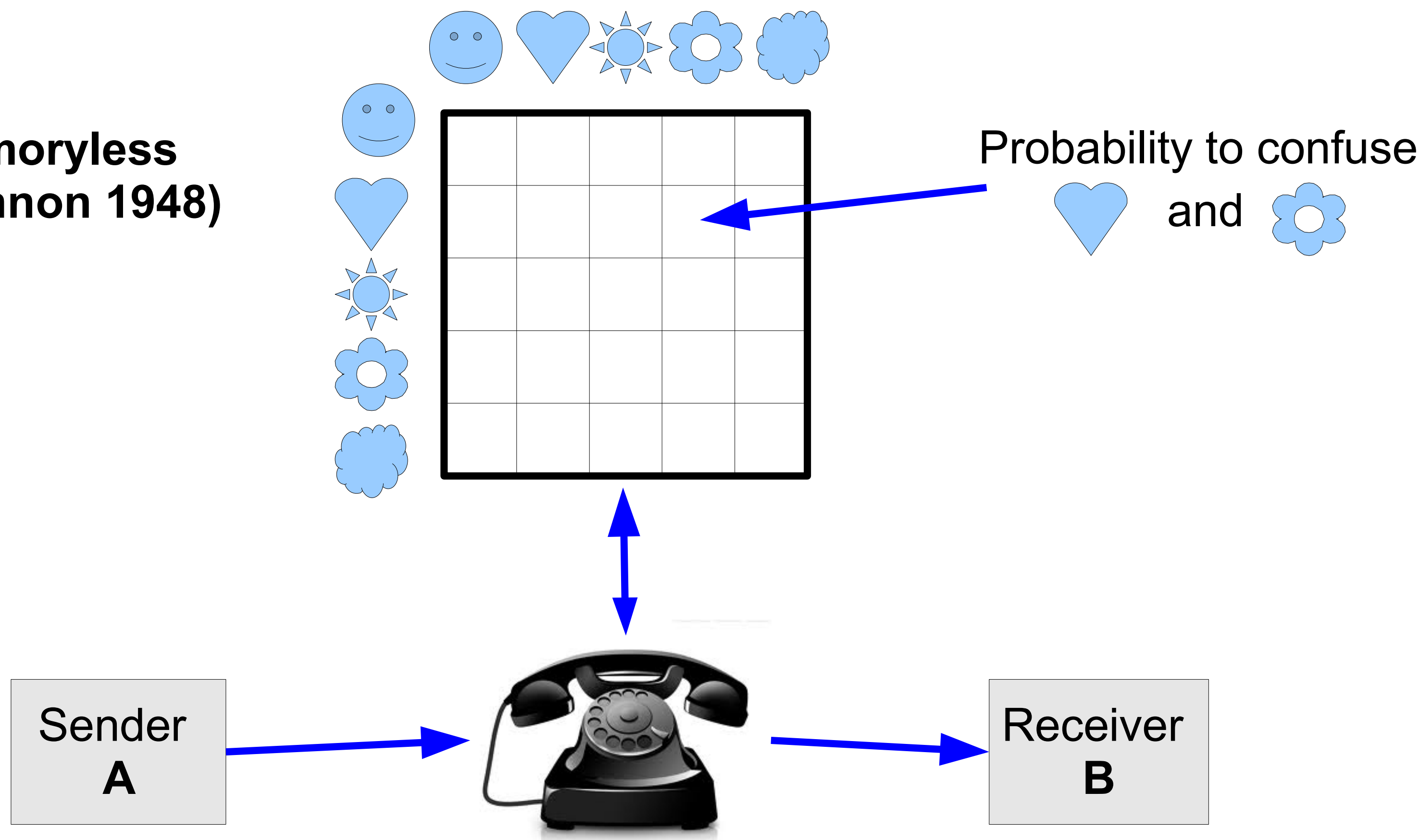
2

Abstract networks

Simone Severini

Channels

Discrete memoryless Channel (Shannon 1948)



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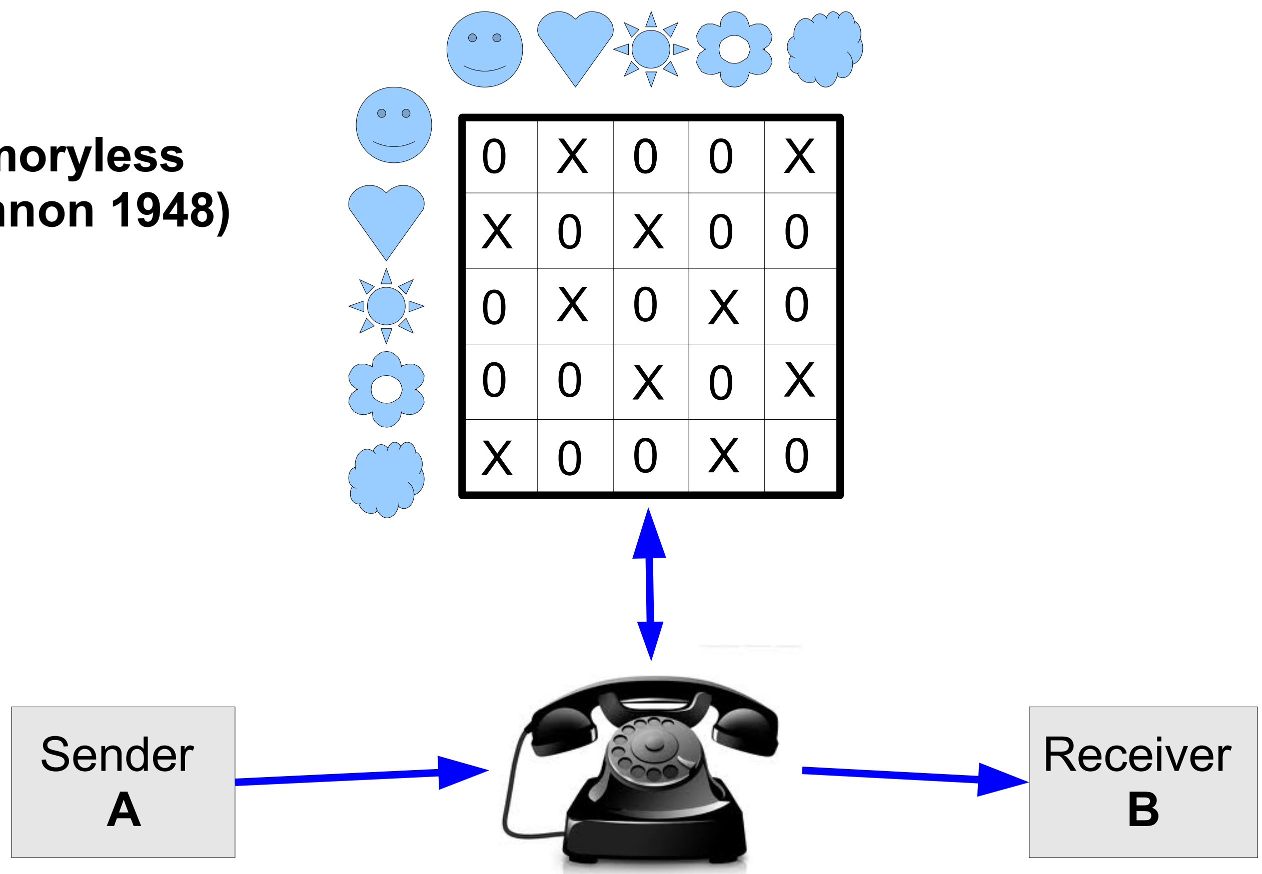
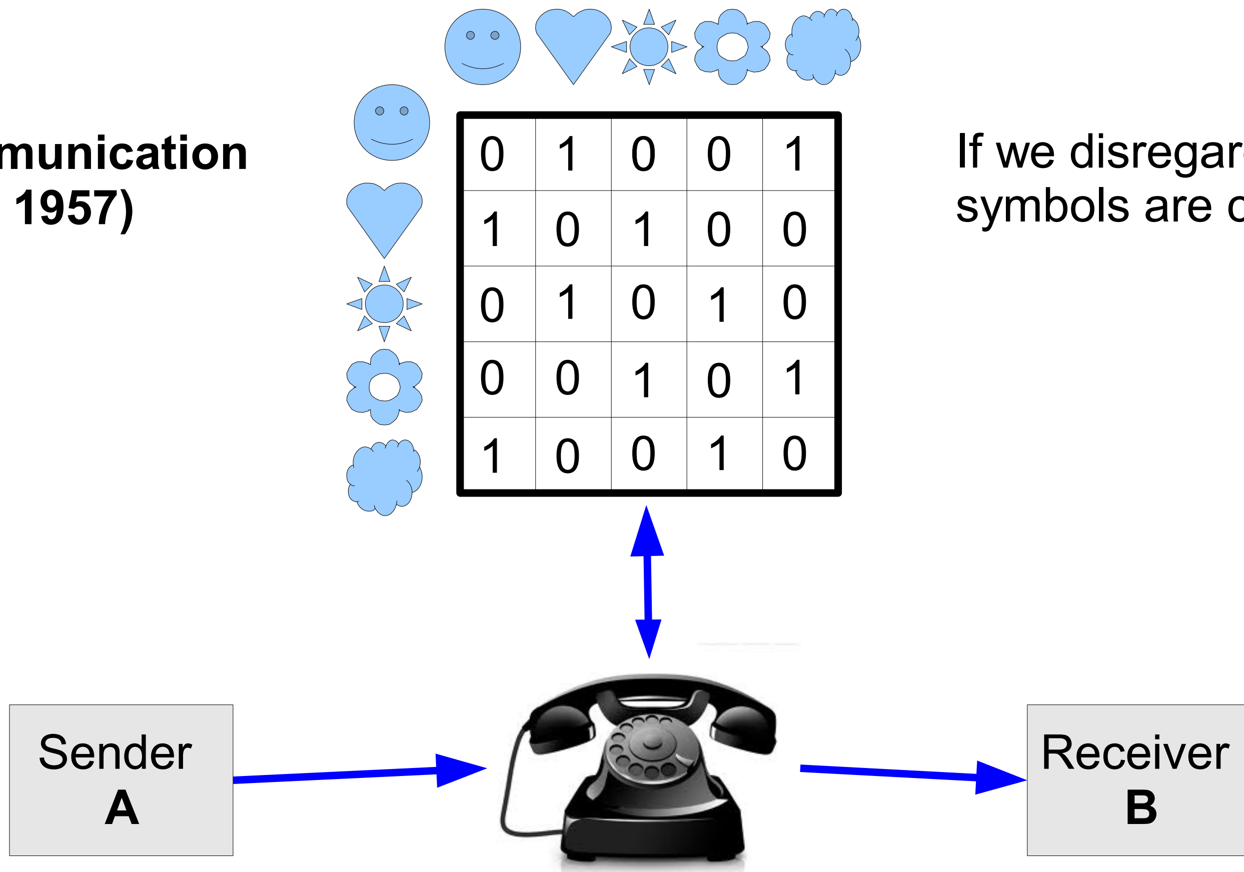


Zero-error communication

Zero-error communication (Shannon 1957)

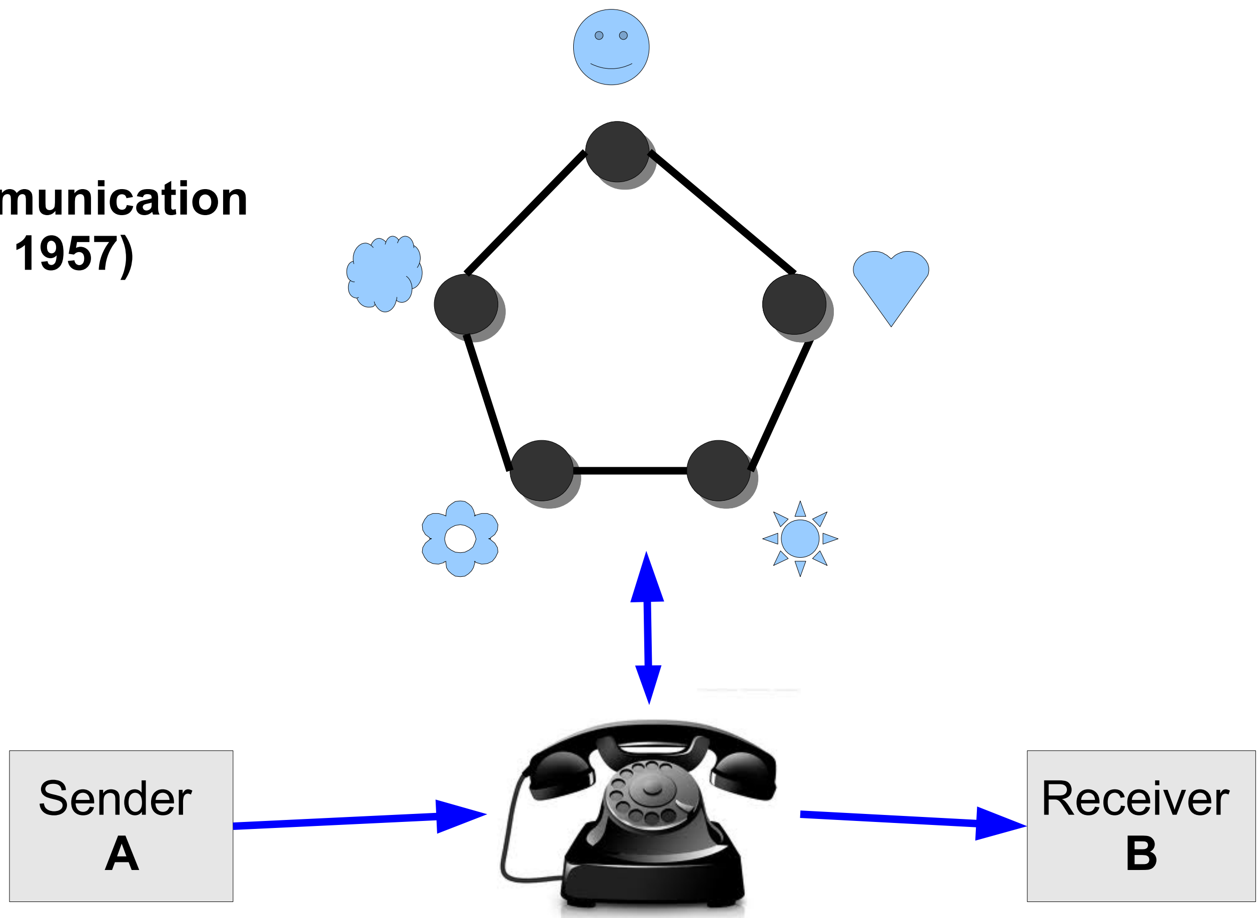
0	1	0	0	1
1	0	1	0	0
0	1	0	1	0
0	0	1	0	1
1	0	0	1	0



If we disregard how much the symbols are confusable...

Zero-error communication

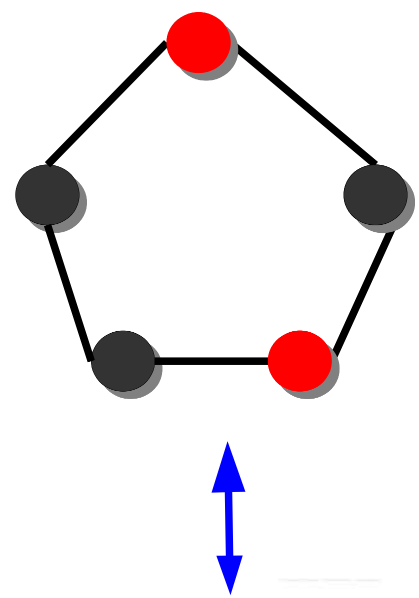
Zero-error communication (Shannon 1957)



Zero-error communication

Zero-error communication (Shannon 1957)

The single-shot zero-error capacity is the independence number, i.e., the max number of non-confusable symbols.

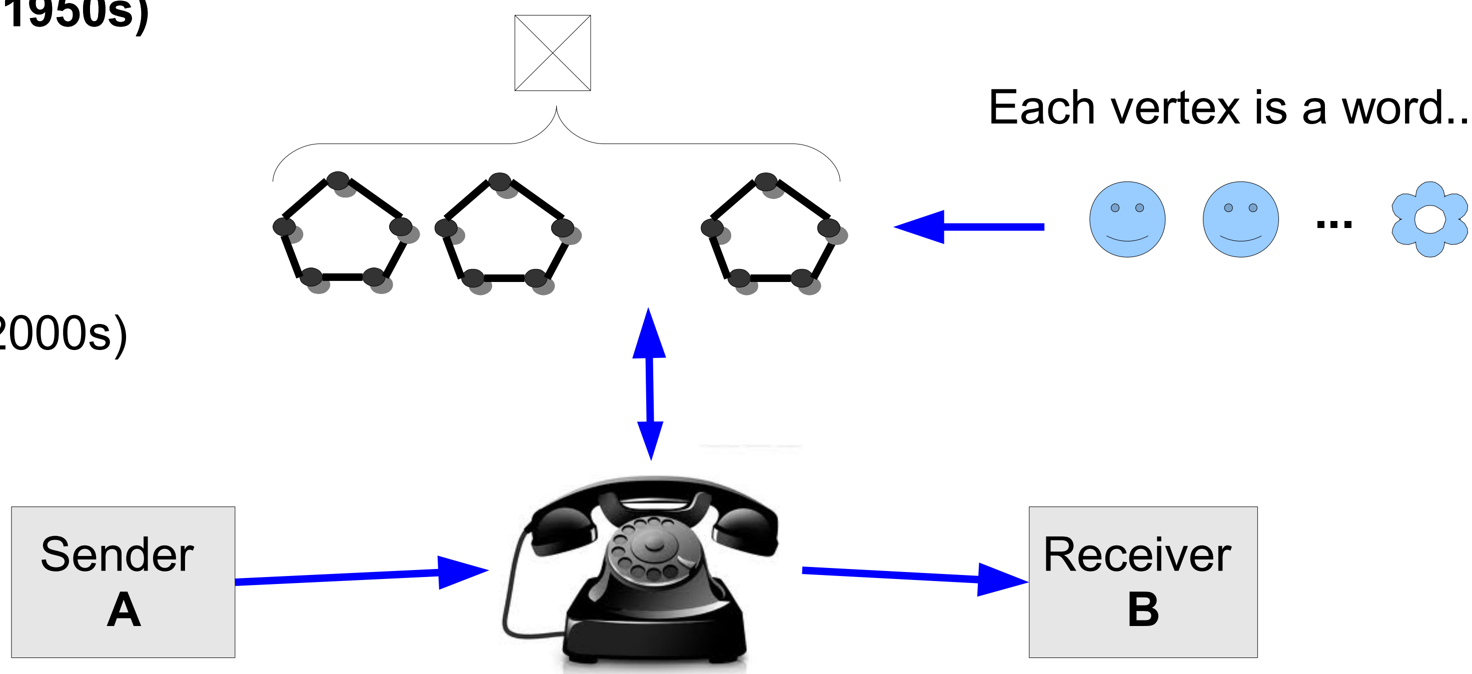


Shannon capacity

Shannon capacity:
This is the max independence number of an infinite series of strong graph products (normalized).

Zero-error communication (Shannon 1950s)

Berge (1970s)
Lovász (1978)
Haemers (1979)
Alon (1990s)
Alon-Lubetsky (2000s)



Lovász number

Zero-error communication (Shannon 1950s)

Berge (1970s)
Lovász (1978)
Haemers (1979)
Alon (1990s)
Alon-Lubetsky (2000s)

1	X	1	1	X
X	1	X	1	1
1	X	1	X	1
1	1	X	1	X
X	1	1	X	1

Lovász number:
The Shannon capacity is not even known to be computable, but it is upper bounded by the largest eigenvalue of symmetric matrices of this form, minimized over their set.

Great conceptual leap!
It keeps into account any possible weight on the edges.

Sender
A

Receiver
B

Generalizations of graphs

Operator system (classical case):

$$S=\{T:\ \forall\{x,y\}\notin E\ \ xTy^{T}=0\}$$

Sender
A

Receiver
B

Shared quantum resources

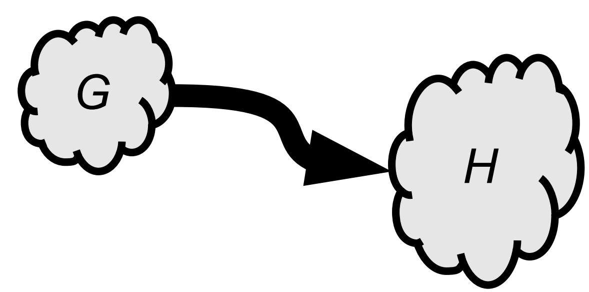
Quantum system
A

Quantum system
B

With quantum resources:

1. The Shannon capacity can increase (there are examples)!
2. We define new graph-theoretic quantities (e.g., min ranks).
3. We generalize graphs and lift the combinatorics to operator theory.

Continuous "deformation"



R. Duan, S. Severini, A. Winter, arXiv:1002.2514v2 [quant-ph]

Summary 2

News:

1. A generalized paradigm for zero-error communication
2. (Again) New network-theoretic parameters
3. A larger information theory related to networks
4. New mathematical tools from operator theory

Problems:

1. (Again) What is the role of network structure?
2. (Again) What about classifying networks according to their new information theoretic properties?
3. Can we define inherently quantum properties of networks (*e.g.,* for the ones for which the quantum-resources-assisted Shannon capacity increases)?

3

Even more abstract networks (examples)

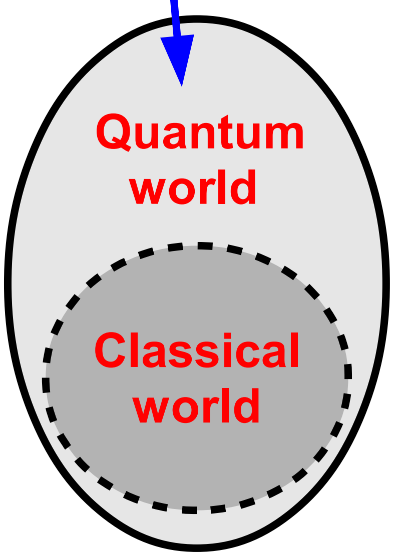
Mathematical representations

Space as a time-dependent graph

Graph Laplacians are quantum states

Unitary matrices aid to hear graphs

Everything classical is also quantum

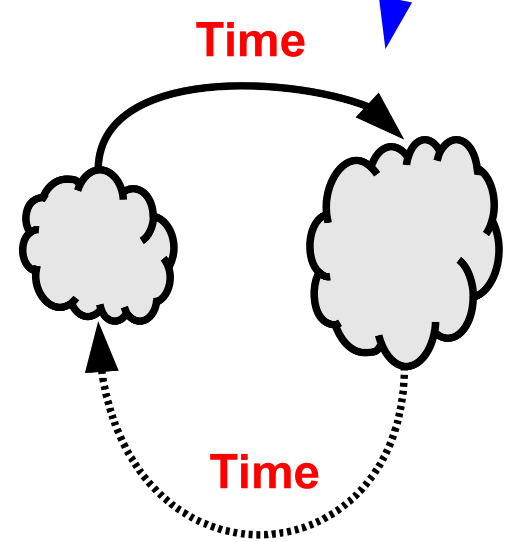


Mathematical axioms

Physical states are matrices generalizing probability distributions

Composite systems need a tensor product

Dynamics is reversible



Simone Severini

Laplacians are quantum states

Laplacian:

$$L(G) = \Delta(G) - A(G)$$

Degree matrix   Adjacency matrix

$$[\Delta]_{i,j} = d(i)\delta_{ij}$$

Laplacians are symmetric;
all the eigenvalues are nonnegative

$$\rho(G) = \frac{1}{Tr(\Delta)} L(G)$$

This is a density matrix
(*i.e.*, a quantum state)

$$\rho(G) = \frac{1}{Tr(\Delta)} \sum_{\{i,j\}\in E} \frac{1}{2}([\boldsymbol{e}_i] - [\boldsymbol{e}_j])$$

Mixture of pure state (projectors)

$$\Psi(G) = \sum_{\{i,j\}\in E} f_{ij}(\boldsymbol{e}_i - \boldsymbol{e}_j) \otimes \boldsymbol{e}_{ij} \in \boldsymbol{H}_V \otimes \boldsymbol{H}_E$$

Purification

$$S(G) = -\sum_i \lambda_i \log \lambda_i$$

Von Neumann Entropy:
It is the measure of "entanglement" between "vertices and edges".

S. L. Braunstein, S. Ghosh, S. Severini, *Ann. Comb.*, **10**:3, 2006

Unitary matrices aid to hear graphs

Graph isomorphism:

$$A(G) = P\,A(H)\,P^T$$

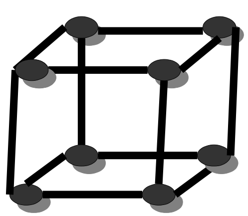
$$G \simeq H$$

Permutation matrix

$$G \simeq H \Rightarrow Sp(G) = Sp(H)$$

Spectrum

"Can one hear the shape of a drum?" was posed by Marc Kac (1966)



Not always...

$$\forall (i,j),(k,l) \in D(G)$$

Biorientation

$$[U(G)]_{(i,j),(k,l)} = 2/d(j) - \delta_{il},\ j=k\,;\,0$$

Grover matrix (it's unitary)

Algorithm:

$$[X(G,p)]_{x,y} = 1 \Leftarrow [U^p(G)]_{i,j} > 0$$
$$[X(G,p)]_{x,y} = 0 \Leftarrow [U^p(G)]_{i,j} \leqslant 0$$

$$G \simeq H \Leftrightarrow Sp(X(G,3)) \neq Sp(X(H,3))$$

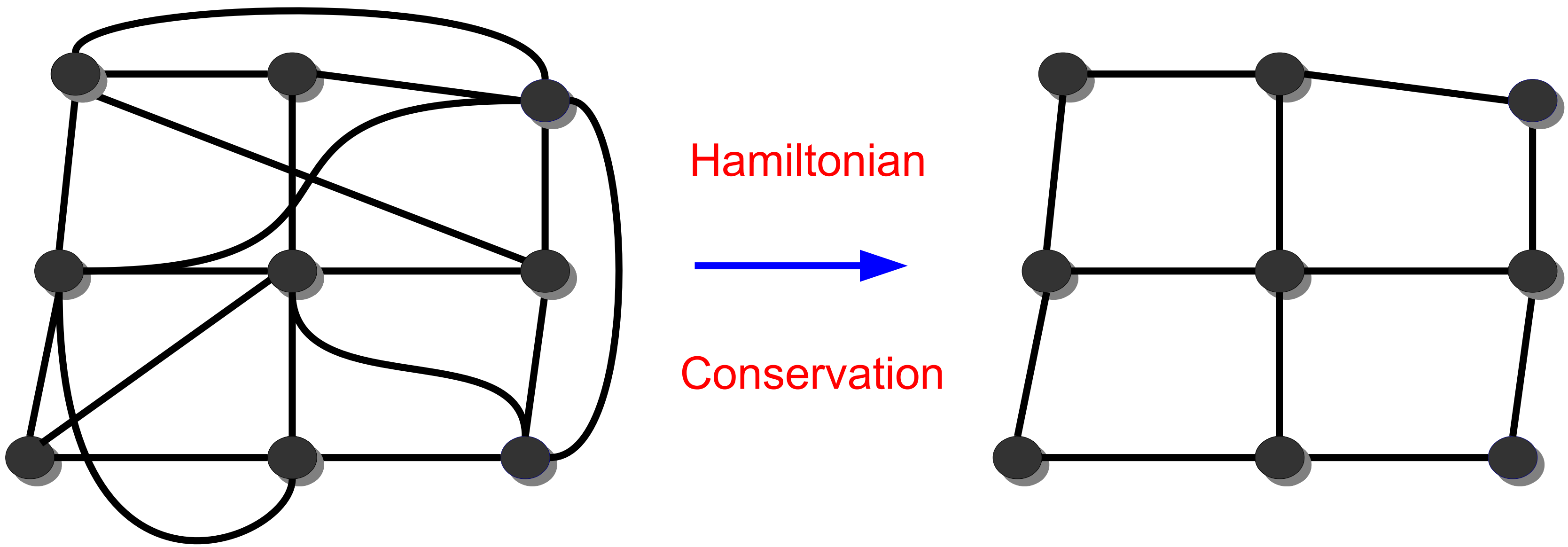
For all regular graphs up to 13 vertices

Conjectured to hold for all strongly regular graphs (tested for up to 64 vertices)

D. Emms, E. Hancock, S. Severini, R. Wilson, *Pattern Recognition* **42**(9): (2009)

Space as a time-dependent graph

Wheeler's geometrogenesis



The early universe was hot:
no geometry

Later on, at lower temperature:
geometry

T. Konopka, F. Markopoulou, S. Severini, *Phys.Rev. D* **77**:104029, (2008)

Networks & quanta

information theory

statistical mechanics

Network coding

Complex networks

Graphs as channels

Background independent models of gravity

Quantum colouring

Graph states

Isomorphism (via encoding)

Quantum walks

Graphs of unitary matrices

Quantum expanders

State transfer on spin systems

Complexity metrics

combinatorics

computational complexity

A new conference in 2013?

Quantum Information and Graph Theory: emerging connections, *Perimeter Institute for Th. Phys., 2008*